Exponential Functionals of Levy Processes: Mellin Transforms and Tail Asymptotics

Bert Zwart (TU Eindhoven)

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Abstract

Let $X(t)$ be a Levy process converging to $-\infty$ a.s. and let Z be its associated exponential functional, i.e. $Z = \int_0^\infty \exp X(t)dt$. This functional appears in many applications, especially mathematical finance, in which Z is often called a perpetuity.

After giving some more motivating applications, the problem of obtaining the Mellin transform of Z is investigated. It is shown that the Bohr-Mollerup characterization of the Gamma function is very useful.

After this, the right (i.e. not left) tail asymptotics of Z are presented. It is shown that, depending on the nature of the Levy process X, the tail of Z can range from extremely heavy $(1/\log x)$ to extremely light $(\exp -x^p, p > 1)$. The proof techniques are mostly probabilistic, but differ from case to case. For the Cramér case, we present a new proof based on a relationship between Levy processes and self-similar Markov processes.

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