Gaussian fluctuation of Young diagrams in the bulk

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Abstract

In this talk we will report a recent work on Gaussian fluctuations of Young diagrams under the Plancherel measure.

Let $\mathcal{P}_n$ be the set of all partitions of n, denote by P_n the Plancherel measure. Let $\lambda = (\lambda_1, \lambda_2, \cdots,)$ be a partition of n. We are mainly interested in the asymptotic behaviour of λ as $n \to \infty$. The limit shape of λ was studied by Logan and Shepp, independently by Vershik and Kerov as early as in the seventies; while the fluctuation of λ at the edge was described by the Airy ensemble as a remarkable achievement in the nineties.

Just recently we prove that for given $0 < x < 2$ the $\lambda_{\lceil \sqrt{n}x \rceil}$, suitably scaled, satisfies the central limit theorem, where $\lceil \sqrt{n}x \rceil$ stands for the least integer greater than $\sqrt{n}x$. More precisely,

$$\frac{\lambda_{\lceil \sqrt{n}x \rceil} - \sqrt{n}\omega(x)}{\frac{1}{2\pi}\sqrt{\log n}} \to N(0, \sigma_x^{-2}), \qquad \text{as } n \to \infty$$

where $\omega(x)$ is the limit curve and $\sigma_x^2 = \frac{1}{\pi^2} \arccos^2 \frac{\omega(x) - x}{2}$.

Our proof is based on the standard poissonization and depoissonization techniques and the Costin-Lebowitz central limit theorem for the number of particles in an interval under the determinantal structure. The computation directly uses the classical results on asymptotics of Bessel functions in different regions, although laborious.

References

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