BRANCHING PROCESSES IN RANDOM ENVIRONMENT

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Branching processes in random environment (BPRE's) are one of the most interesting generalizations of the Galton-Watson branching processes. We deal here with the simplest case of BPRE's when the sequential states of the environment are independent and identically distributed. This model admits the following informal description. Let $f_n(s)$ be the (random) offspring generating function of particles of the n-th generation. Denote by Z_n the number of individuals in the n-th generation of the BPRE. In the notation above the evolution of our BPRE is described by the relations

$$Z_0 = 1, \ \mathbf{E}\left[s^{Z_{n+1}} \mid f_0, f_1, ..., f_n; Z_0, Z_1, ..., Z_n\right] = \left(f_n(s)\right)^{Z_n}.$$

Let $X_k = \log f_k'(1)$, $S_0 = 0$, $S_n = X_1 + \ldots + X_n$, $n \geq 1$. We call a BPRE critical if the following Spitzer's condition holds:

$$\frac{1}{n}\sum_{k=1}^{n} P(S_k > 0) \to \rho \in (0,1), \quad n \to \infty.$$

The most important characteristics of a BPRE are the asymptotic behavior of its survival probability and the number of particles in the process as $n \to \infty$. Another interesting characteristics of a branching process is its genealogical tree. It may be described to a certain extent by the so-called reduced branching process $\{Z_{m,n}, \ 0 \leq m \leq n\}$ in which $Z_{m,n}$ is equal to the number of particles in the process $\{Z_k, \ 0 \leq k \leq n\}$ at time $m \leq n$ each of which has a non-empty offspring at time n. In this talk we give a survey of the recent results for critical BPRE related with the asymptotic behavior of the survival probability and functional limit theorems for Z_n and demonstrate a number of limit theorems for reduced BPRE's. Quenched and annealed approaches will be discussed.

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