Estimation Under Shape Constraints: Monotone, Convex, and Beyond

Talk to be given at Utrecht
Wednesday, December 8, 2004

by

Jon A. Wellner
Department of Statistics, University of Washington, Seattle WA
(visiting Vrije Universiteit, Amsterdam)

Abstract: In this talk I will consider estimation of an unknown density function f under shape constraints from a mixture model perspective. Let k be a non-negative integer and let G be a distribution function on $(0, \infty)$. Then, with $x_+ = x1\{x > 0\}$,

$$f(x) = \int_0^\infty \frac{1}{y} \left(1 - \frac{x}{ky}\right)_+^{k-1} dG(y)$$

is monotone (decreasing) when $k = 1$, g is convex and decreasing when $k = 2$, and higher values of k correspond to densities which are $k - 1$ times differentiable with derivatives of alternating sign. When $k \to \infty$ the limiting form of the family is

$$f(x) = \int_0^\infty \frac{1}{y} \exp(-x/y) dG(y)$$

corresponding to a *completely monotone* density. I will discuss what is known concerning maximum likelihood estimation of f and the mixing distribution G when $k = 1$, $k = 2$, and $k = \infty$, and then discuss current work connected with the cases $3 \leq k < \infty$. Splines and a particular Hermite interpolation problem play a role.
(Based on joint work with Fadoua Balabdaoui.)