Representation of Fixed Points of a Smoothing Transformation

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Given a sequence of real random variables $T = (T_1, T_2, ...)$, distributions μ which satisfy the following fixed point equation for distributions are considered:

$$W \stackrel{\sim}{=} \sum_{j=1}^{\infty} T_j W_j \ ,$$

where $W, W_1, W_2, ...$ have distribution μ and $T, W_1, W_2, ...$ are independent. These solutions are known to arise, e.g., in the limiting behaviour of branching processes. Here, such fixed points are characterized as mixtures of infinitely divisible distributions. Depending on the properties fo T and of the fixed points in question, it can be shown that the corresponding infinitely divisible distributions can be normal, α-stabel ($\alpha < 1$) or degenerate.