Trapping of sediment in tidal estuaries

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Estuary: semi-enclosed body of water where salt and fresh water meet.
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Chesapeake Bay: Length: 315 km
Width: 5-56 km
Av. depth: 8.5 m

German Wadden Sea:
Many estuaries exhibit an **Estuarine Turbidity Maximum** comprising fine, suspended muddy sediments.

1. Potomac  
2. Chesapeake Bay  
3. Delaware  
4. Severn
Other example: Ems estuary.

**Ems River at a glance**

~ 12,600 km²

$Q_{avg} \sim 70 \text{ m}^3/\text{s}$
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Our focus: Ems Estuary

Shipping most Important Industry

e.g. MeyerWerft

Large Implications for river And estuarine dynamics!
**Ems River at a glance**

- Area: $\sim 12,600 \text{ km}^2$
- Average discharge: $Q_{\text{avg}} \sim 70 \text{ m}^3/\text{s}$

**Our focus:** Ems Estuary

- Shipping most important industry
  - e.g. MeyerWerft
- Large implications for river and estuarine dynamics!
This resulting in
1. a significant change in tidal motion (safety)
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Changes in horizontal velocity?
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2. a significant increase of turbidity (environment)
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Introduction

This resulting in:

1. a significant change in tidal motion (safety)
2. a significant increase of turbidity (environment)

- Turbidity maximum has moved upstream;
- High turbidity zone now extended into the freshwater zone to Papenburg;
• Can the observed changes in the water motion be modelled and understood?
• Which mechanisms result in trapping of sediment in the Ems and what has changed over the years?
Research Questions

- Can the observed changes in the water motion be modelled and understood?
- Which mechanisms result in trapping of sediment in the Ems and what has changed over the years?

Main Results

Essential ingredients

- Decrease of bed friction and vertical mixing and a deepening of the channel
- Along-estuary varying erosion coefficient (~ layer of fine sediment)
- Temporal settling lag effects + external overtides
**Geometry:**
- weakly convergent
- prescribed (fixed) bed

**Forcing:**
- sea side: $M_2$ and $M_4$ water elevation
- river side: fresh water flux
- prescribed density gradient
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- weakly convergent
- prescribed (fixed) bed

Forcing:
- sea side: $M_2$ and $M_4$ water elevation
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- prescribed density gradient
• Water Motion: 2 DV (width averaged) shallow water equations (residual, $M_2$ and $M_4$ components)
Model Formulation (2)

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\[ u_t + uu_x + wu_z + g\zeta_x - \frac{g\rho_x}{\rho_0}(z - \zeta) - (A\nu u_z)_z = 0. \]

+ appropriate boundary conditions
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\[
\begin{align*}
\frac{u_t}{u_{x}} + w_z - \frac{u}{L_b} &= 0, \\
\frac{u_{t}}{u_{x}} + uu_{x} + wu_{z} + g\frac{\zeta}{\rho_0}(z - \zeta) - (A_vu_z)_z &= 0.
\end{align*}
\]

+ appropriate boundary conditions

\[
\rho(s) = \rho_0(1 + \beta < s(x) >)
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\[ \zeta(t, x) = A_{M_2} \cos \sigma t + A_{M_4} \cos(2\sigma t - \phi) \]

\[ B(x) \int_{-H}^{\zeta} u \, dz = Q \quad \text{at} \quad x = L. \]

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In principle $\rho$ depends on sediment concentration as well!

\[
\rho(s) = \rho_0(1 + \beta <s(x)>)
\]
Model Formulation (3)

- Water Motion: 2 DV (width averaged) shallow water equations (residual, $M_2$ and $M_4$ components)
- Suspended load transport:
  - advection-diffusion equation
  - deposition
  - erosion $\sim a(x) |u|$

Erosion flux: 
\[ E_s = -K_v \frac{\partial c}{\partial z} n_z - K_h \frac{\partial c}{\partial x} n_x = w_s c_* \]

Deposition flux: 
\[ D = w_s c n_z \]
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\[ c_*(t, x) = \frac{\rho_s}{\rho_0 g' d_s} \frac{|\tau_b(t, x)|}{a(x)} \]
Model Formulation (4)

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\[
(1-p) \rho_s z_b = - \nabla \cdot q_s
\]

Convergence: increase of $z_b$  Divergence: decrease of $z_b$
### Model Formulation (4)

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### Diagnostic in density

- **Bed evolution:**

\[
(1-p) \, \delta_T \, z_b = - \nabla \cdot q_s
\]

With the flux $q_s = \left\langle \int_H (\zeta \, (uc - K_h c_x)) \, dz \right\rangle$
Solution Method (1)

Analytical solution method

Perturbation approach: physical variables are expanded in power series of a small parameter $\varepsilon = AM2/H$. 
Analytical solution method

Velocities $u$ and $w$
Concentration $C$

Shallow water equations

$$u = u^{02} + \varepsilon(u^{10} + u^{14}) + \ldots$$

Concentration equation

$$c = c^{00} + c^{04} + \ldots + \varepsilon(c^{10} + c^{12} + \ldots)$$
Analytical solution method

Velocities u and w
Concentration C

Residual sediment transport, that still depends on the erosion coefficient $a(x)$

\[
\int_{-H}^{0} (u^{10} c^{00} + \langle u^{02} c^{12} \rangle + \langle u^{14} c^{04} \rangle - K_h \langle c_x^{00} \rangle) dz + \langle \zeta^0 [u^0 c^0]_{z=0} \rangle
\]
Residual sediment transport, that still depends on the erosion coefficient $a(x)$

- Assume morphodynamic equilibrium: no residual sediment transport

$$T(x) a(x) + F(x) a_x(x) = 0$$

With $T(x)$:
- $\sim <u> <C>$: residual contribution
- $\sim <u_{M2} C_{M2}>$: settling lag ($M_2$)
- $\sim <u_{M4} C_{M4}>$: tidal asymmetry ($M_4$)
- diffusive contribution
- sum of all terms

With $F(x)$:
- $\sim <K_h C>$: settling lag
Experiments

- Two years are considered: 1980 and 2005.
- Most parameters are obtained from observations.
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![Graphs showing M2 tidal amplitude in 1980 and 2005](image)

- Vertical Mixing: 0.0187 m²/s
- Bottom Friction: 0.0124 m²/s
- Vertical Mixing: 0.098 m/s
- Bottom Friction: 0.049 m/s
Experiments

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• Furthermore we choose:
  River outflow = 70 m$^3$/s
  Settling velocity = 2 mm/s

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Using this information, we can solve for the unknown erosion coefficient:

$$T(x) \ a(x) + F(x) \ a_x(x) = 0$$
Experiments

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- Furthermore we choose:
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\[ T(x) a(x) + F(x) a_x(x) = 0 \]

This gives the sediment trapping in the estuary.
Model parameters:
• River discharge 70 m³/s;
• Setting velocity 2 mm/s.
Model Results (5)

Model parameters:
- River discharge 70 m$^3$/s;
- Setting velocity 2 mm/s.

2005:
1) ETM has shifted far upstream by 32 km;
2) ETM is located in the fresh water zone.
Maximum of sediment concentration coincides with zeros of transport function $T$.

\[
\text{as } T(x) \ a(x) + F(x) \ a_x(x) = 0
\]
Maximum of sediment concentration coincides with zeros of transport function $T$.

\[\text{(as } T(x) \ a(x) + F(x) \ a_x(x) = 0)\]

So to understand the changes in the trapping location of the sediment, we have to inspect the function $T$ and its different contributions more carefully:
Model Results (7)

\[ T = \int_{-H}^{0} \frac{u^{10} c^{00}}{a} \, dz + \left\langle \zeta \left[ \frac{u^{0} c^{0}}{a} \right]_{z=0} \right\rangle + \int_{-H}^{0} \left\langle \frac{u^{02} c^{12}}{a} \right\rangle \, dz + \int_{-H}^{0} \left\langle \frac{u^{14} c^{04}}{a} \right\rangle \, dz - \int_{-H}^{0} K_h \left\langle \frac{c^{00}}{a} \right\rangle_{x} \, dz. \]

Components of the transport function \( T \):

- \[ \int_{-H}^{0} \frac{u^{10} c^{00}}{a} \, dz + \left\langle \zeta \left[ \frac{u^{0} c^{0}}{a} \right]_{z=0} \right\rangle \text{ Residual transport function} \]
- \[ \int_{-H}^{0} \left\langle \frac{u^{02} c^{12}}{a} \right\rangle \, dz \text{ M}_2 \text{ transport function} \]
- \[ \int_{-H}^{0} \left\langle \frac{u^{14} c^{04}}{a} \right\rangle \, dz \text{ M}_4 \text{ transport function} \]
- \[ \int_{-H}^{0} K_h \left\langle \frac{c^{00}}{a} \right\rangle_{x} \, dz \text{ Diff. transport func.} \]
Model Results (3)

\[ T = \int_{-H}^{0} u^{10} c^{00} \frac{d}{dz} + \left\langle \zeta \left[ u^{0} c^{0} \right]_{z=0} \right\rangle + \int_{-H}^{0} \left\langle u^{02} c^{12} \right\rangle d{z} + \int_{-H}^{0} \left\langle u^{14} c^{04} \right\rangle d{z} - \int_{-H}^{0} K_h \left\langle \frac{c^{00}}{a} \right\rangle d{z}. \]

Components of the transport function \( T \):

- Residual transport function
  \[ \int_{-H}^{0} u^{10} c^{00} \frac{d}{dz} + \left\langle \zeta \left[ u^{0} c^{0} \right]_{z=0} \right\rangle \]

- \( M_2 \) transport function
  \[ \int_{-H}^{0} \left\langle u^{02} c^{12} \right\rangle d{z} \]

- \( M_4 \) transport function
  \[ \int_{-H}^{0} \left\langle u^{14} c^{04} \right\rangle d{z} \]

- Diff. transport func.
  \[ \int_{-H}^{0} K_h \left\langle c^{00} \right\rangle d{z} \]

Gravitational circulation

Tidal return flow

River inflow

Tidal stresses

Surface contribution
Maximum of sediment concentration coincides with zeros of transport function $T$. The convergence point has moved upstream mainly due to change of the $M_2$ contribution.
Maximum of sediment concentration coincides with zeros of transport function $T$.

The convergence point has moved upstream mainly due to change of the $M_2$ contribution.
The behavior of the $T_{M2}$ component has changed due to changes in the externally prescribed $M_4$:

**TIDAL ASYMMETRY MECHANISM**
CHANGE OF TRAPPING LOCATION DUE TO CHANGE IN TIDAL ASYMMETRY (CHARACTER OF M4 TIDAL WAVE)
Model Results (10)

Dependency on grain size (1980)

Increasing grain size
Model Results (10)

Dependency on grain size (1980)

Position insensitive to grain size

Increasing grain size
Model Results (11)

Dependency on grain size (2005)

Increasing grain size
Model Results (11)

Dependency on grain size (2005)

Position very sensitive to grain size

Increasing grain size
Model Results (12)

Dependency on river outflow (2005)

Increasing River outflow
Model Results (13)

Dependency on river outflow (2005)

Increasing River outflow
Model Results (14)

Extending the model by making density depend on Sediment concentration as well

Results in a nonlinear differential equation for the sediment availability $a(x)$
Model Results (15)

Results for the 1980 case

Increasing Sediment concentration

Width increases
Model Results (16)

Results for the 2005 case

Increasing Sediment concentration

Width increases considerably
Conclusions

• Formation of ETMs can be modelled with an idealised model. Essential Ingredients:
  • Along-estuary *varying* erosion coefficient
    (~ layer of fine sediment)
  • Tidal asymmetry

• Physical mechanisms resulting in the ETMs can be understood using the idealised model.

• Difference in trapping of sediment in Ems Estuary mainly a result of the changes in tidal asymmetry between 1980 and 2005.

• Model sensitivities to parameters and parameterizations can be easily investigated. The trapping locations in 1980 are not very sensitive to parameter changes, in 2005 the locations change dramatically when changing parameters.