# Trapping of sediment in tidal estuaries

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# Introduction (1)

Estuary: semi-enclosed body of water where salt and fresh water meet.

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Chesapeake Bay: Length: 315 km
Width: 5-56 km
Av. depth: 8.5 m

German Wadden Sea:

# Introduction (2)

Many estuaries exhibit an
EstuarineTurbidity Maximum
comprising fine, suspended
muddy sediments.


1. Potomac
2. Chesapeake Bay
3. Delaware
4. Severn

# Introduction (3)

Other example: Ems estuary.


## Ems River at a glance

~ 12,600 km$^2$

$Q_{avg}$ ~ 70 m$^3$/s

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Shipping most Important Industry

e.g. MeyerWerft

Large Implications for river And estuarine dynamics!

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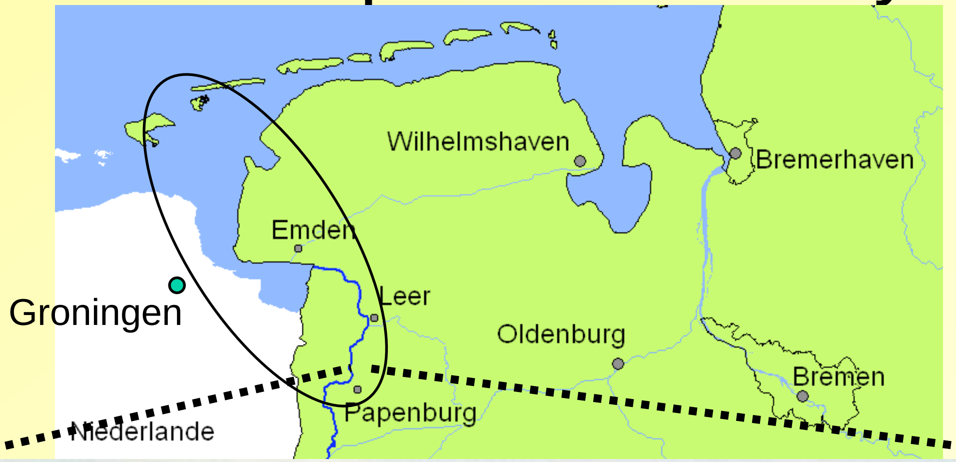


# Ems River at a glance

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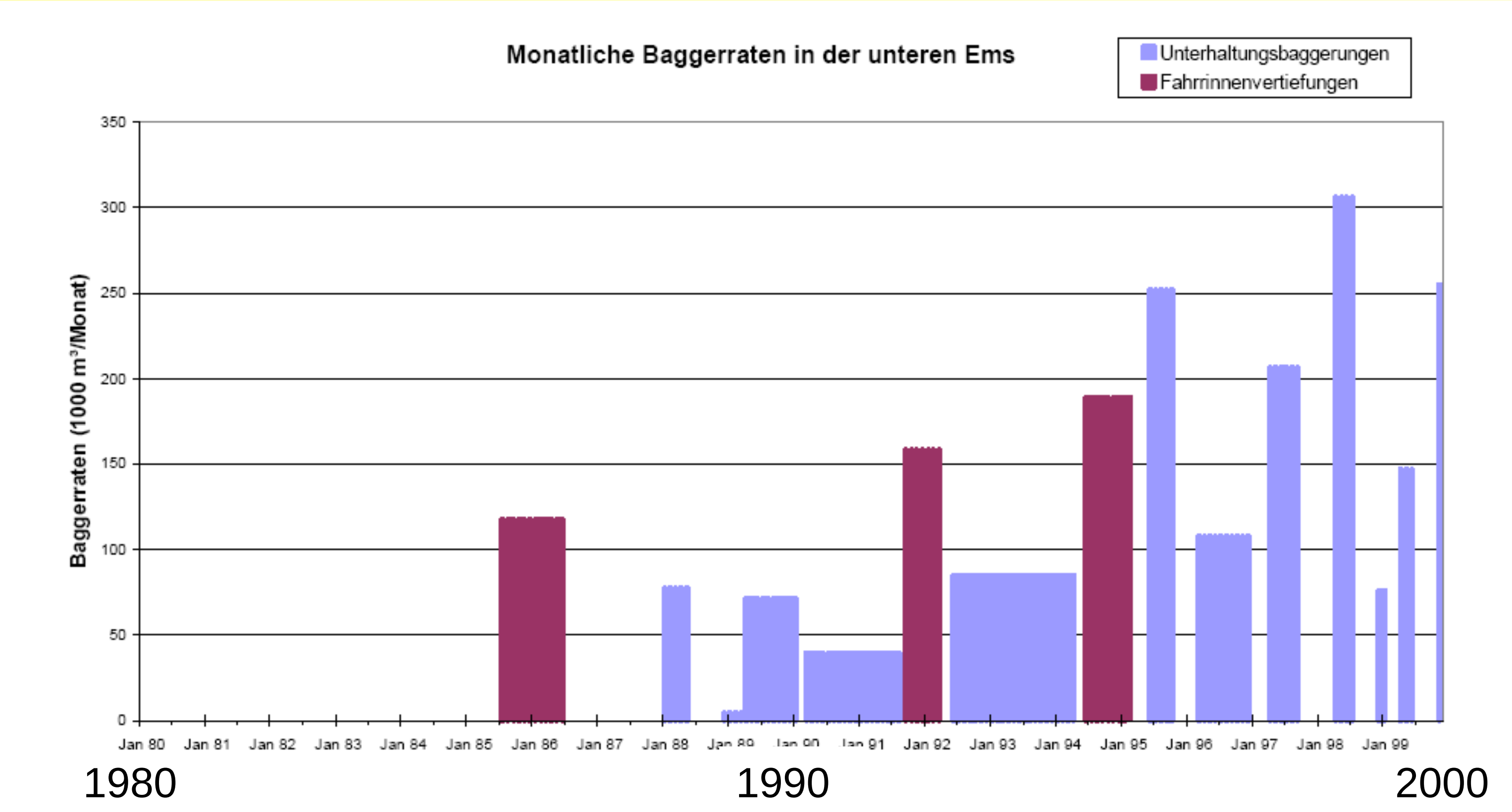
# Introduction (4)

## Increase in Dredging




**Monthly dredging rates in the Ems between Emden and Pappenburg between 1980 and 2000, in units of 1000 m³/month. Adapted from Habermann, 2003.**

# Introduction (5)

This resulting in

1. a significant change in tidal motion (safety)

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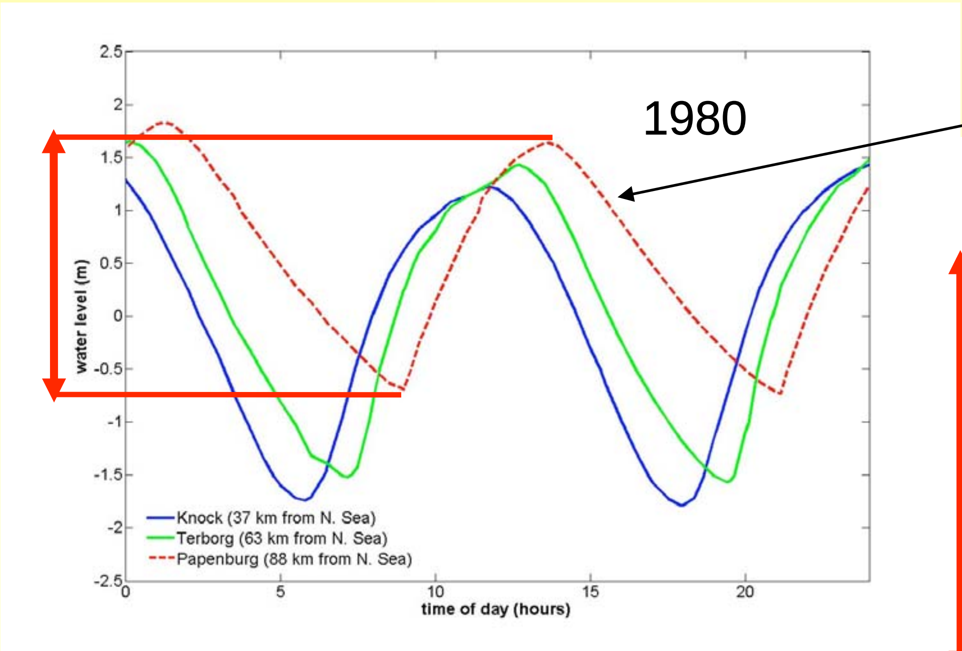
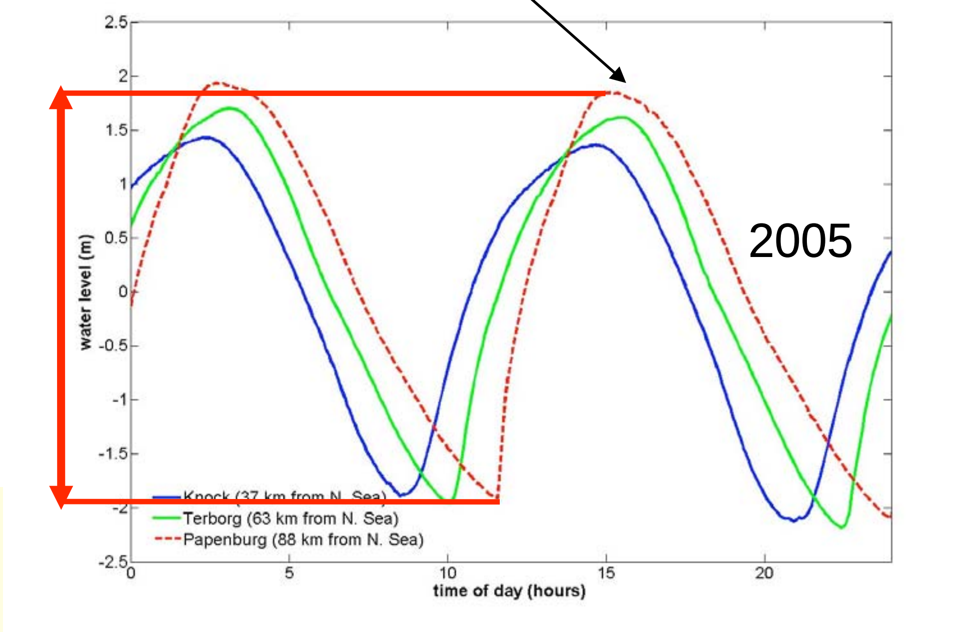
Tidal Range Increased by ~ 1.5 m

1980




2005




**Changes in horizontal velocity?**

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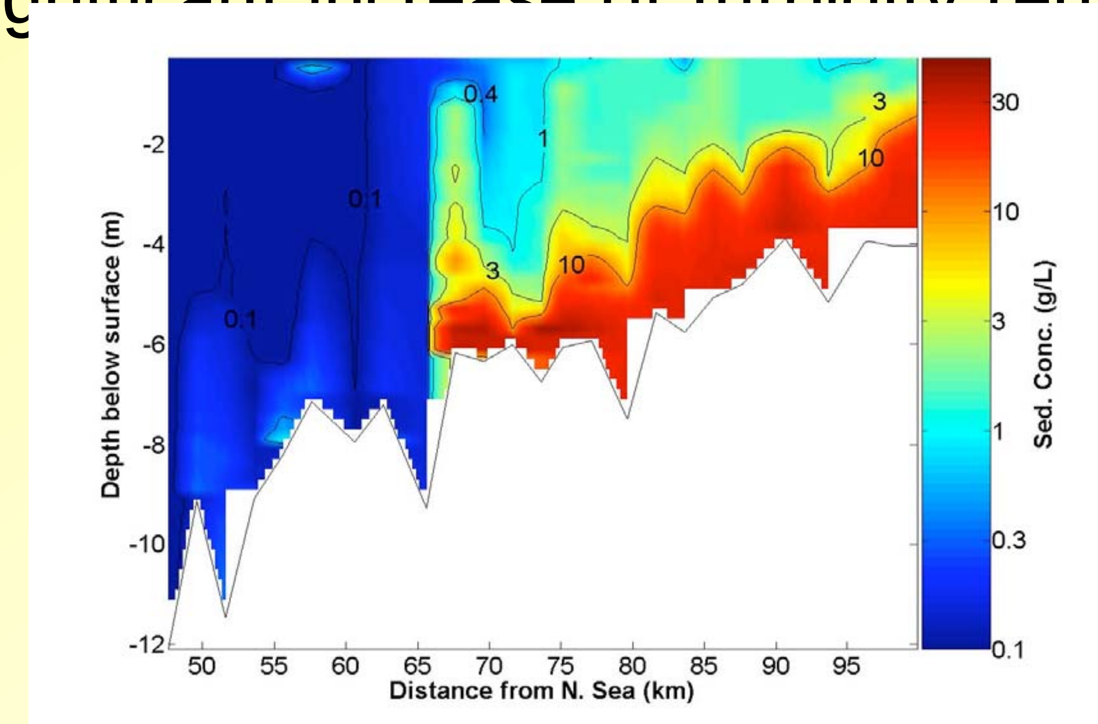


- Turbidity maximum has moved upstream;
- High turbidity zone now extended into the freshwater zone to Papenburg;

# Research Questions

• Can the observed changes in the water motion be modelled and understood?

• Which mechanisms result in trapping of sediment in the Ems and what has changed over the years?

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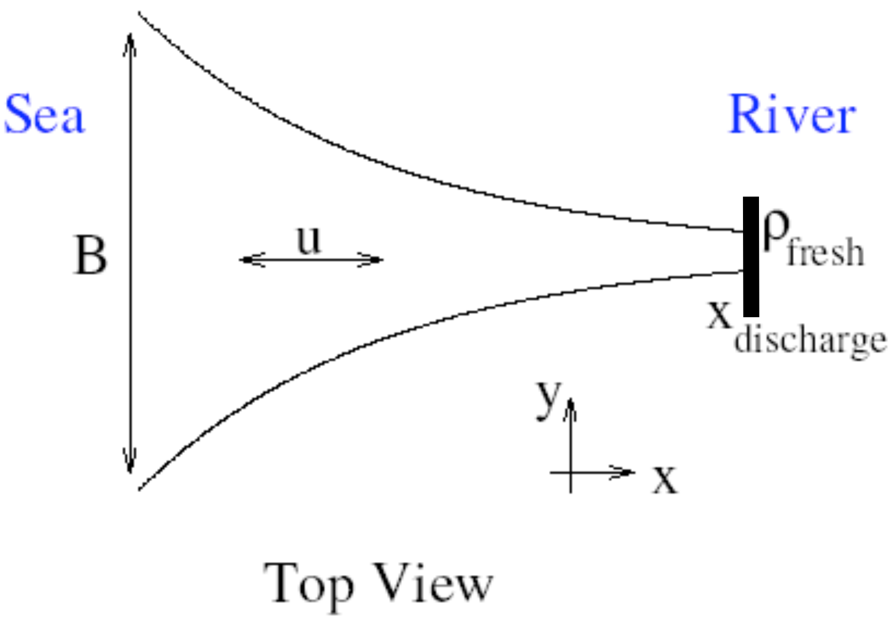
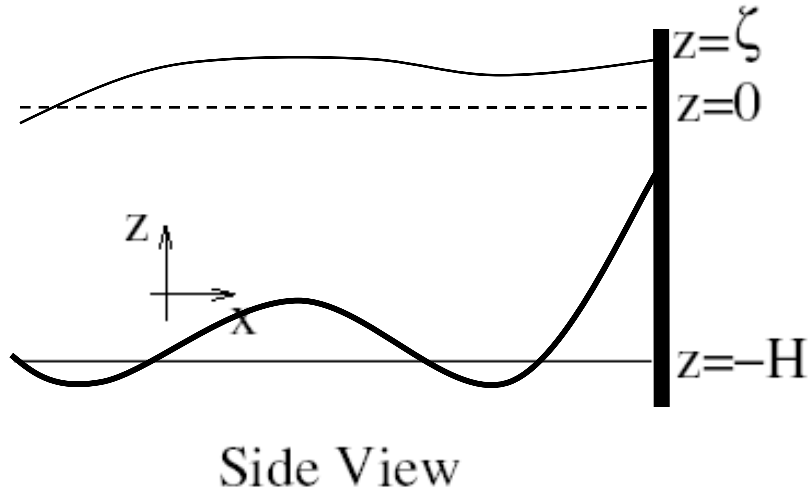
## Main Results

# Essential ingredients

- Decrease of bed friction and vertical mixing and a deepening of the channel
- Along-estuary varying erosion coefficient (~ layer of fine sediment)
- Temporal settling lag effects + external overtides

# Model Formulation (1)




**Geometry:**

- weakly convergent
- prescribed (fixed) bed

**Forcing:**

- sea side: $M_2$ and $M_4$ water elevation
- river side: fresh water flux
- prescribed density gradient

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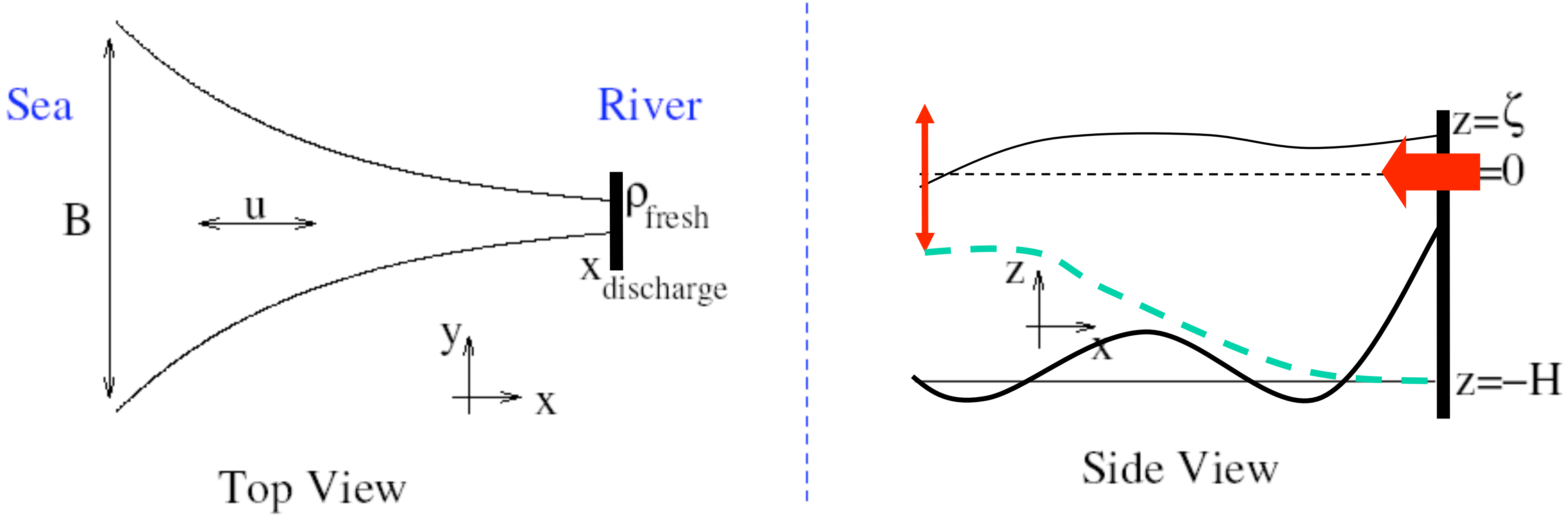


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$$u_x + w_z - \frac{u}{L_b} = 0,$$

$$u_t + uu_x + wu_z + g\zeta_x - \frac{g\rho_x}{\rho_0}(z-\zeta) - (A_v u_z)_z = 0.$$

+ appropriate boundary conditions

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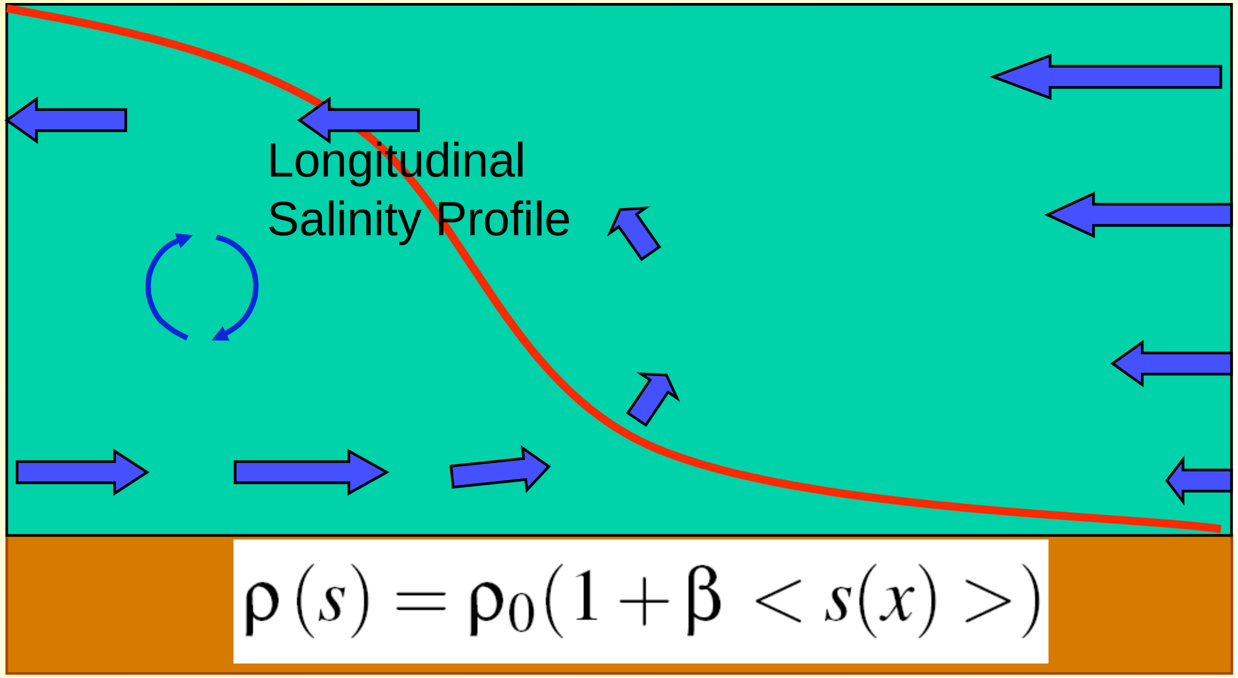


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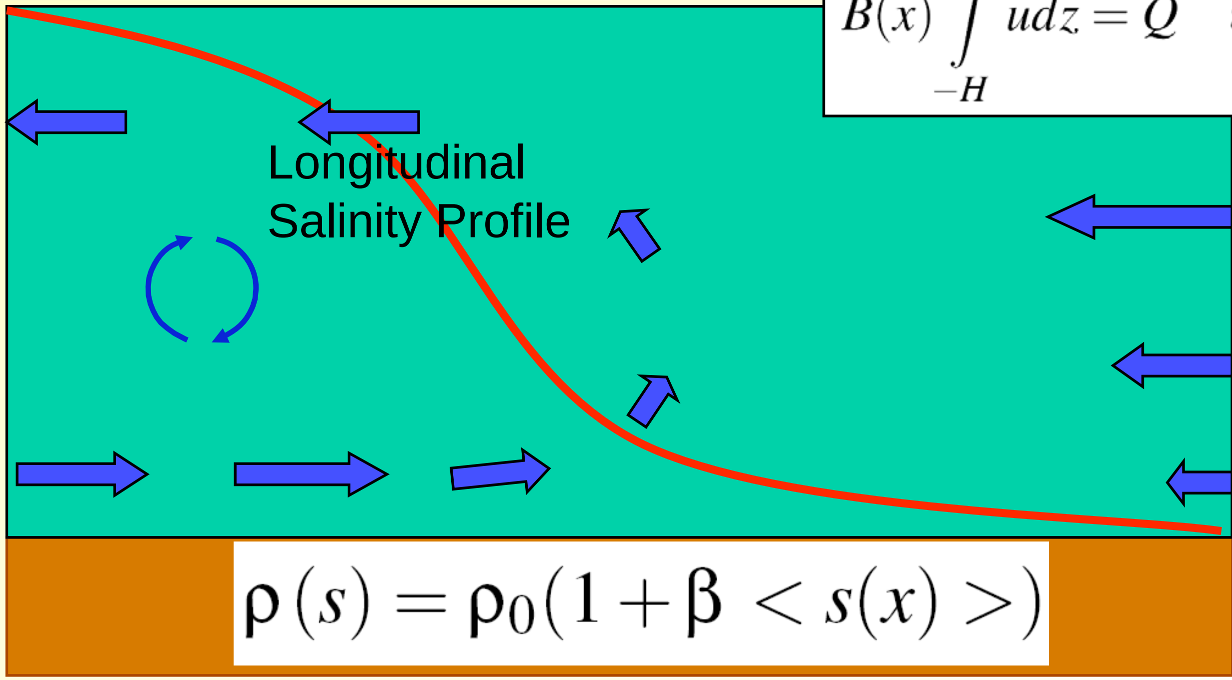
$$\zeta(t,x) = A_{M_2}\cos\sigma t + A_{M_4}\cos(2\sigma t - \phi)$$

$$B(x)\int_{-H}^{\zeta} u\,dz = Q \quad \text{at} \quad x = L.$$

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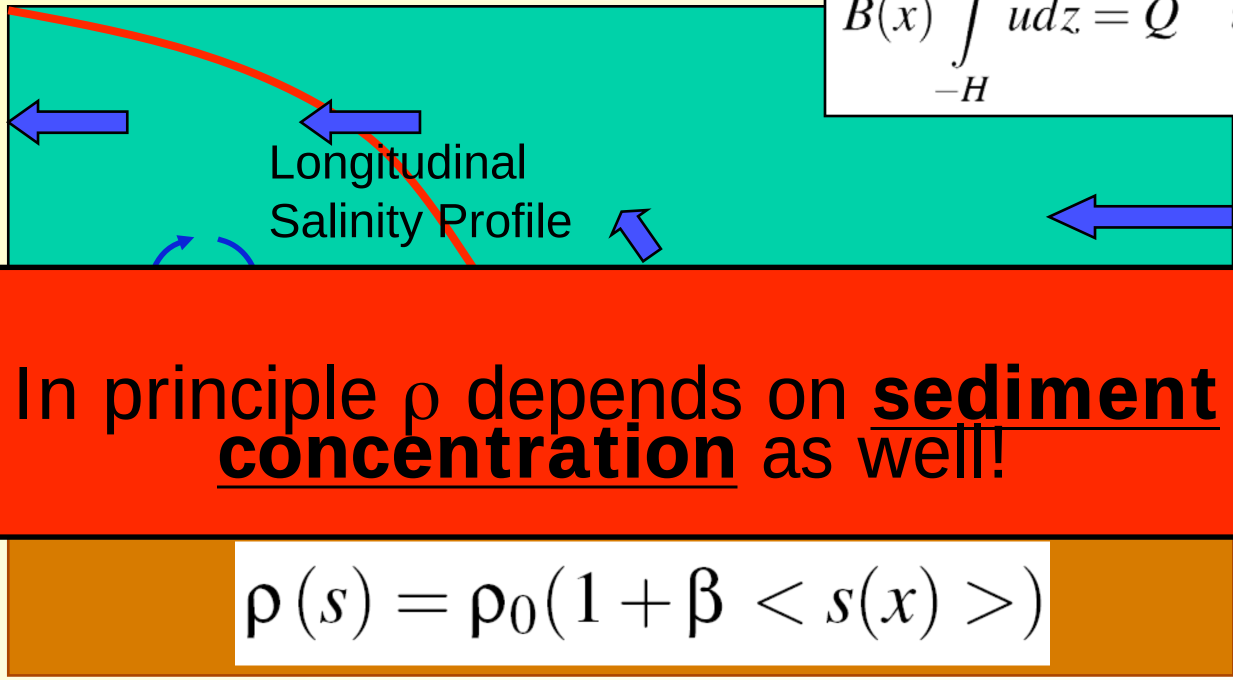


In principle $\rho$ depends on **sediment concentration** as well!

$$\rho(s) = \rho_0(1 + \beta < s(x) >)$$

# Model Formulation (3)

• Water Motion: 2 DV (width averaged) shallow water equations(residual, $M_2$ and $M_4$ components)

• Suspended load transport:

- advection-diffusion equation
- deposition
- erosion ~ a(x) |u|

---

Erosion flux: $$E_s \equiv -K_v \frac{\partial c}{\partial z} n_z - K_h \frac{\partial c}{\partial x} n_x = w_s c_*$$

Deposition flux: $$D = w_s c n_z$$

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$$c_*(t,x) = \rho_s \frac{|\tau_b(t,x)|}{\rho_0 g' d_s} a(x)$$

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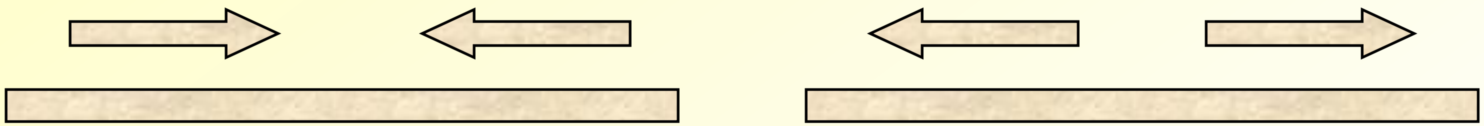
- Diagnostic in density
- Bed evolution:

$$(1\text{-}p)\ \rho_s\ z_b = -\ \nabla\cdot\mathbf{q}_s$$



Convergence: increase of $z_b$ Divergence: decrease of $z_b$

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- Diagnostic in density
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$$(1-p)\, \delta_\tau\, z_b = - \nabla \cdot \mathbf{q}_s$$

With the flux $\mathbf{q}_s$ =

$$\left\langle \int_{-H}^{\zeta} (uc - K_h c_x)\, dz \right\rangle$$

# Solution Method (1)

Analytical solution method

Perturbation approach: physical variables are expanded in power series of a small parameter $\varepsilon$=AM2/H.

# Solution Method (2)

Analytical solution method

Velocities u and w
Concentration C

**Shallow water equations**

$$u = u^{02} + \varepsilon(u^{10} + u^{14}) + \ldots$$

$M_2$ → $u^{02}$

Residual → $u^{10}$

$M_4$ → $u^{14}$

$M_4$: Internal, External

**Concentration equation**

$$c = c^{00} + c^{04} + \ldots + \varepsilon(c^{10} + c^{12} + \ldots)$$

Residual → $c^{00}$

$M_4$ → $c^{04}$

Residual → $c^{10}$

$M_2$ → $c^{12}$

# Solution Method (3)

Analytical solution method

⬇

Velocities u and w
Concentration C

⬇

Residual sediment transport, that still depends on the erosion coefficient a(x)

$$\int_{-H}^{0} (u^{10}c^{00} + \langle u^{02}c^{12}\rangle + \langle u^{14}c^{04}\rangle - K_h \langle c_x^{00}\rangle) dz + \langle \zeta^0 [u^0 c^0]_{z=0} \rangle$$

# Solution Method (4)

**Residual sediment transport, that still depends on the erosion coefficient a(x)**

• Assume morphodynamic equilibrium : no residual sediment transport

$$\Rightarrow \quad T(x)\, a(x) + F(x)\, a_x(x) = 0$$

With T(x):

- (red) $\sim \langle u \rangle \langle C \rangle$ : residual contribution
- (green) $\sim \langle u_{M_2} C_{M_2} \rangle$ : settling lag ($M_2$)
- (black) $\sim \langle u_{M_4} C_{M_4} \rangle$ : tidal asymmetry ($M_4$)
- (pink) diffusive contribution
- (blue) sum of all terms

F(x):

- (orange dashed) $\sim \langle K_h C \rangle$ : settling lag

# Model Results (1)

**Experiments**

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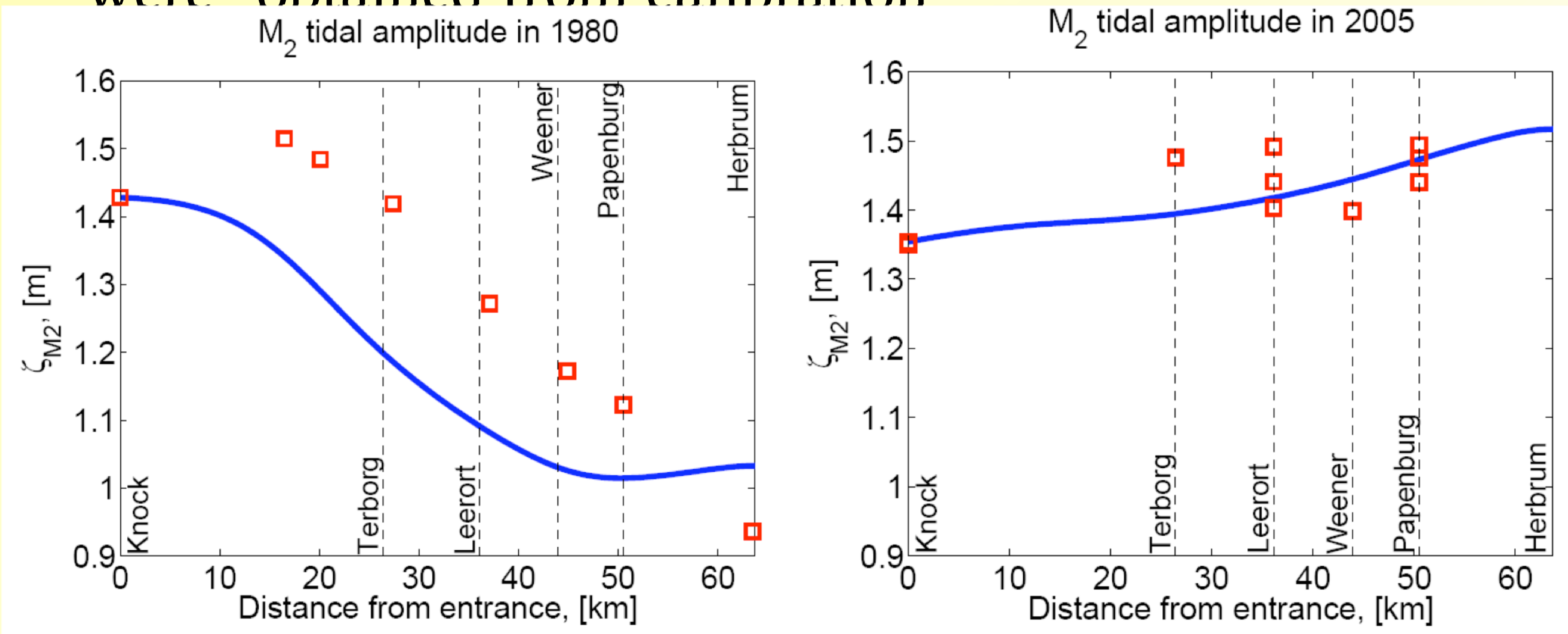
• Only the vertical mixing and bottom friction were  obtained from calibration




| 1980 | | 2005 |
|---|---|---|
| 0.0187 $m^2/s$ | Vertical Mixing | 0.0124 $m^2/s$ |
| 0.098 m/s | Bottom Friction | 0.049 m/s |

# Model Results (3)

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- Furthermore we choose:
  River outflow = 70 $m^3/s$
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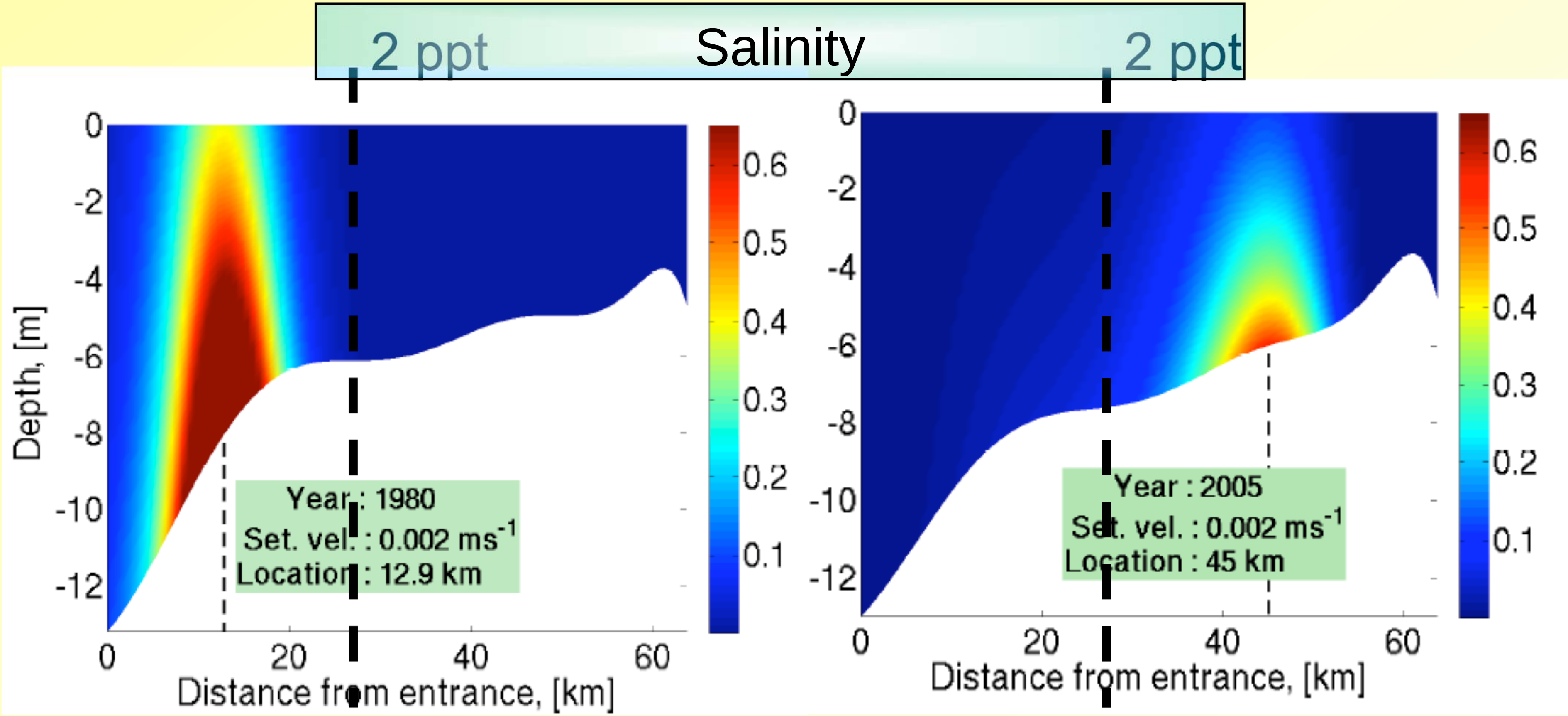
This gives the sediment trapping in the estuary

# Model Results (5)




Model parameters:

- River discharge **70 m³/s;**
- Setting velocity **2 mm/s.**

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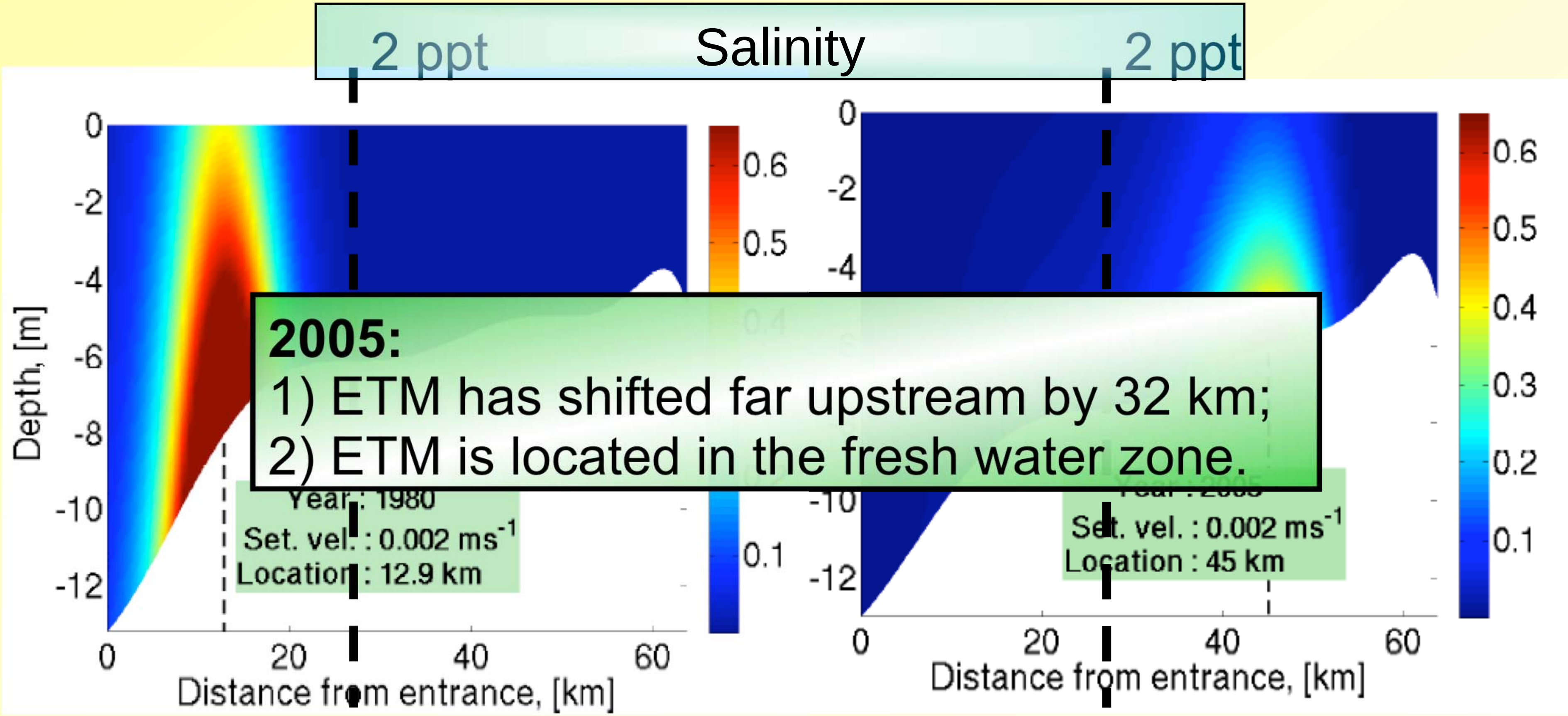


**2005:**
1) ETM has shifted far upstream by 32 km;
2) ETM is located in the fresh water zone.

Model parameters:

- River discharge **70 $m^3/s$;**
- Setting velocity **2 mm/s.**

# Model Results (6)

Maximum of sediment concentration coincides with zeros of transport function T.

(as $T(x)\, a(x) + F(x)\, a_x(x) = 0$)

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So to understand the changes in the trapping location of the sediment, we have to inspect the function T and its different contributions more carefully:

# Model Results (7)

$$T = \underbrace{\int_{-H}^{0} u^{10}\frac{c^{00}}{a}dz + \left\langle \zeta\left[u^{0}\frac{c^{0}}{a}\right]_{z=0}\right\rangle}_{T_{res}} + \underbrace{\int_{-H}^{0}\left\langle u^{02}\frac{c^{12}}{a}\right\rangle dz}_{T_{M_2}} + \underbrace{\int_{-H}^{0}\left\langle u^{14}\frac{c^{04}}{a}\right\rangle dz}_{T_{M_4}} - \underbrace{\int_{-H}^{0} K_h\left\langle \frac{c^{00}}{a}\right\rangle_x dz}_{T_{diff}}.$$

Components of the transport function T:

- (red) $\int_{-H}^{0} u^{10}c^{00}dz + \langle \zeta [u^{0}c^{0}]_{z=0}\rangle$ Residual transport function
- (green) $\int_{-H}^{0} \langle u^{02}c^{12}\rangle dz$ $M_2$ transport function
- (black) $\int_{-H}^{0} \langle u^{14}c^{04}\rangle dz$ $M_4$ transport function
- (magenta) $\int_{-H}^{0} K_h \langle c_x^{00}\rangle dz$ Diff. transport func.

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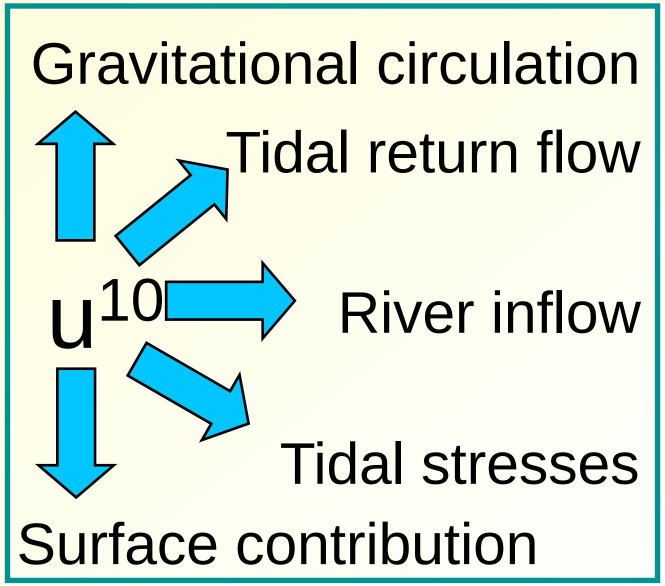
- $\int_{-H}^{0} K_h\langle c^{00}_x\rangle dz$ Diff. transport func.



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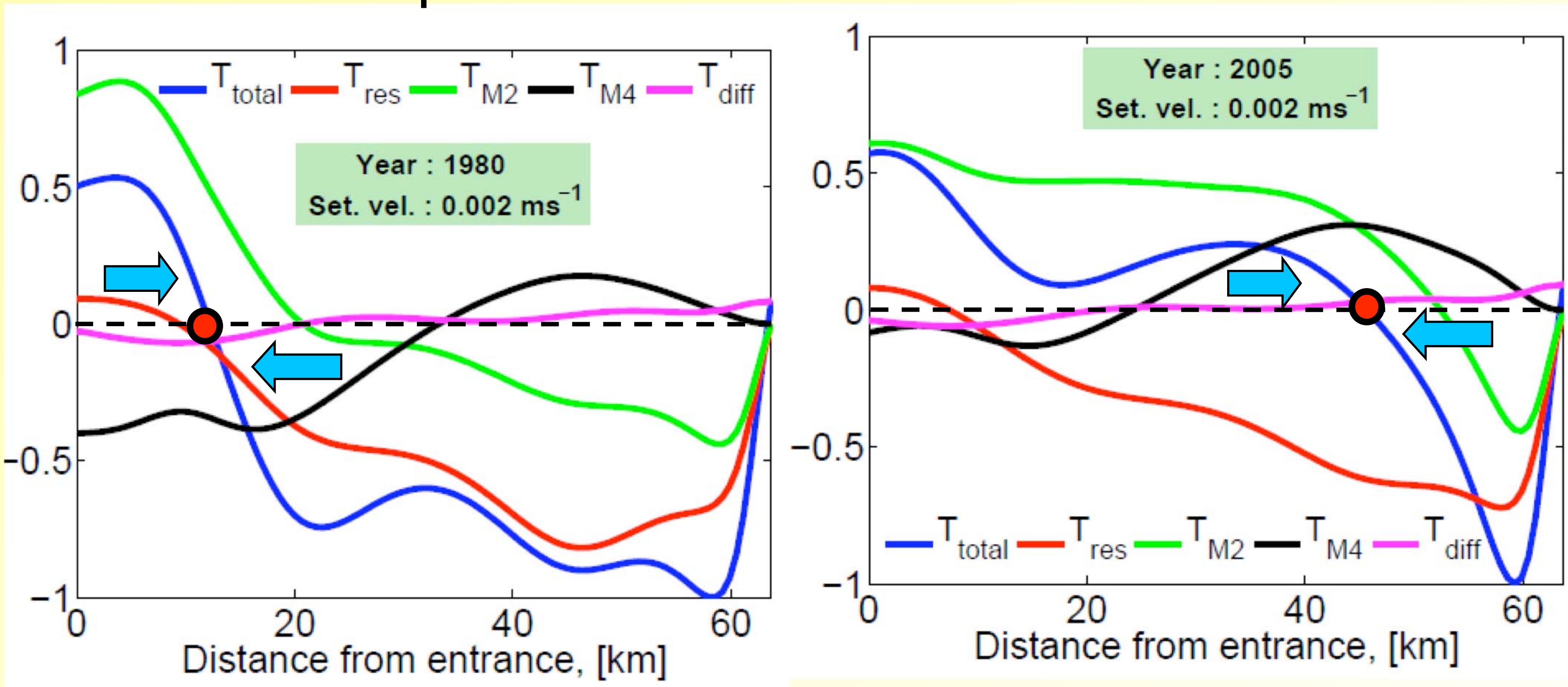


**The convergence point has moved upstream mainly due to change of the $M_2$ contribution**

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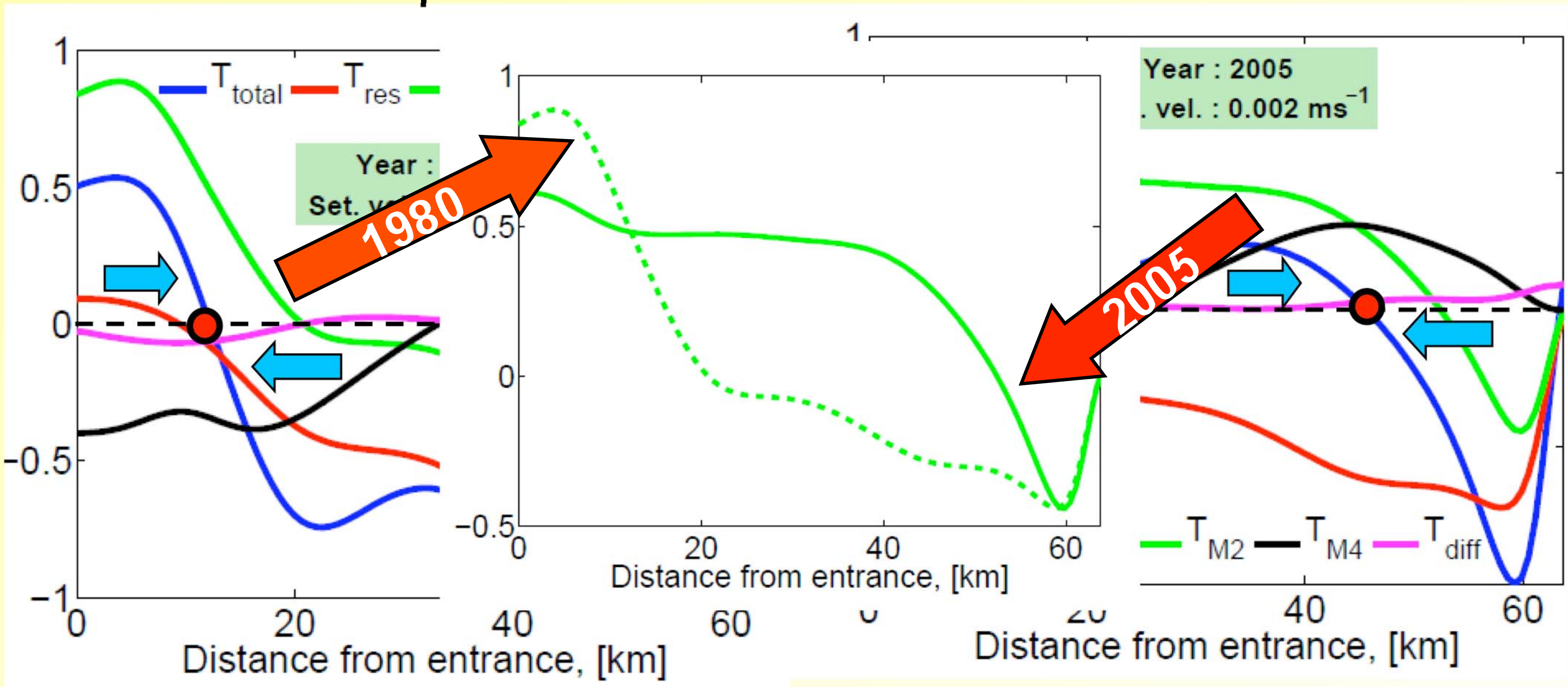


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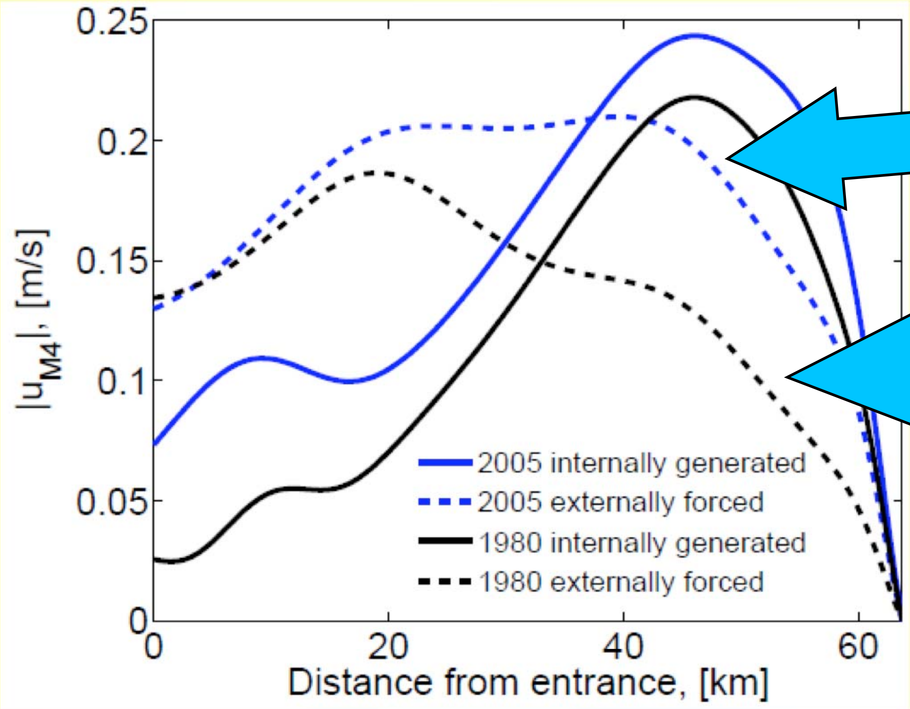
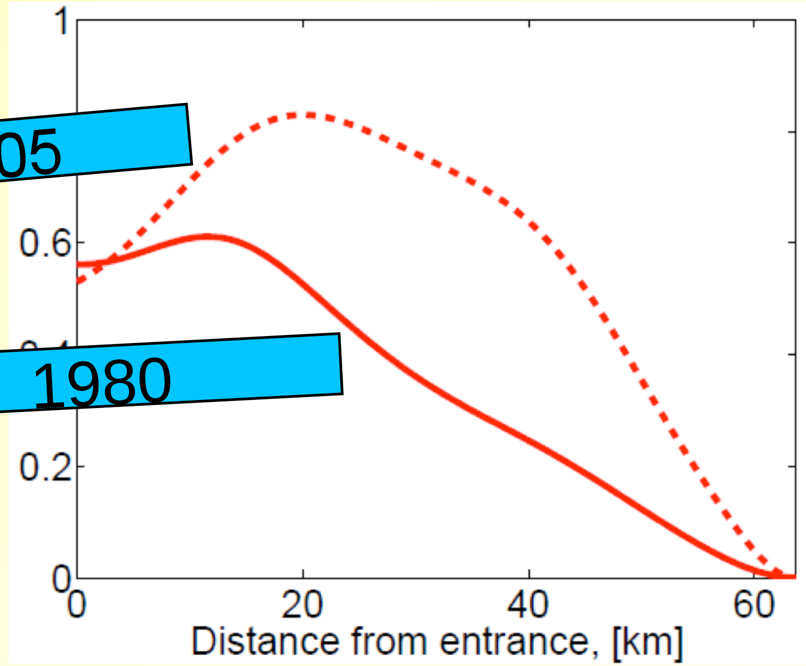
# Model Results (8)

$M_4$ velocity amplitude



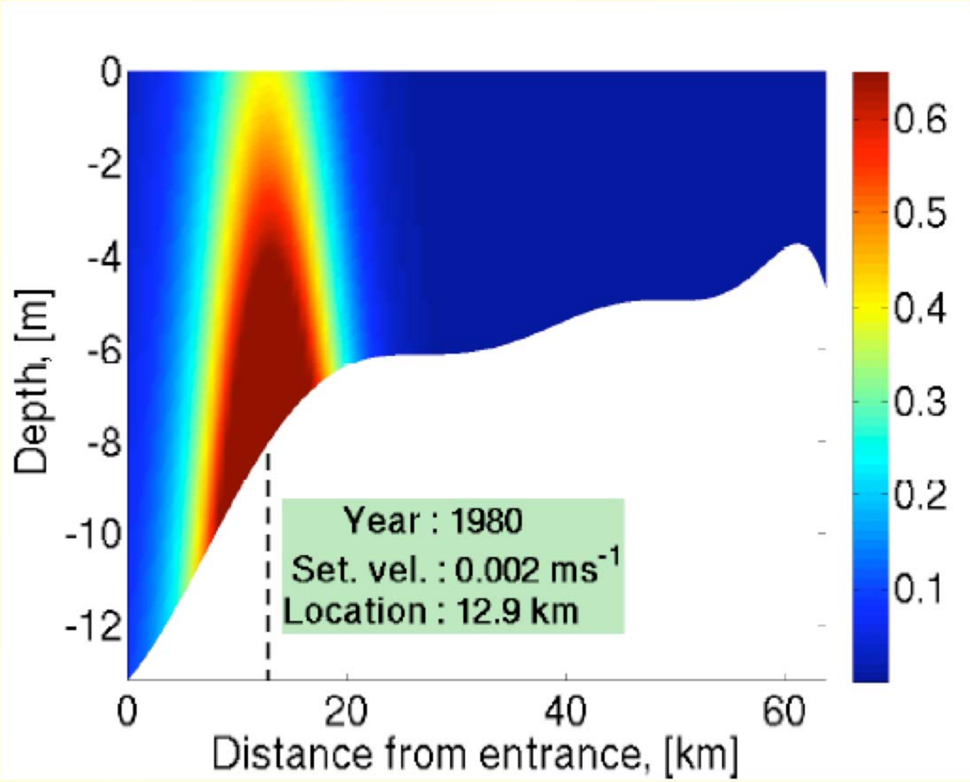

Contribution of $T_{M2}$ due to externally prescribed $M_4$

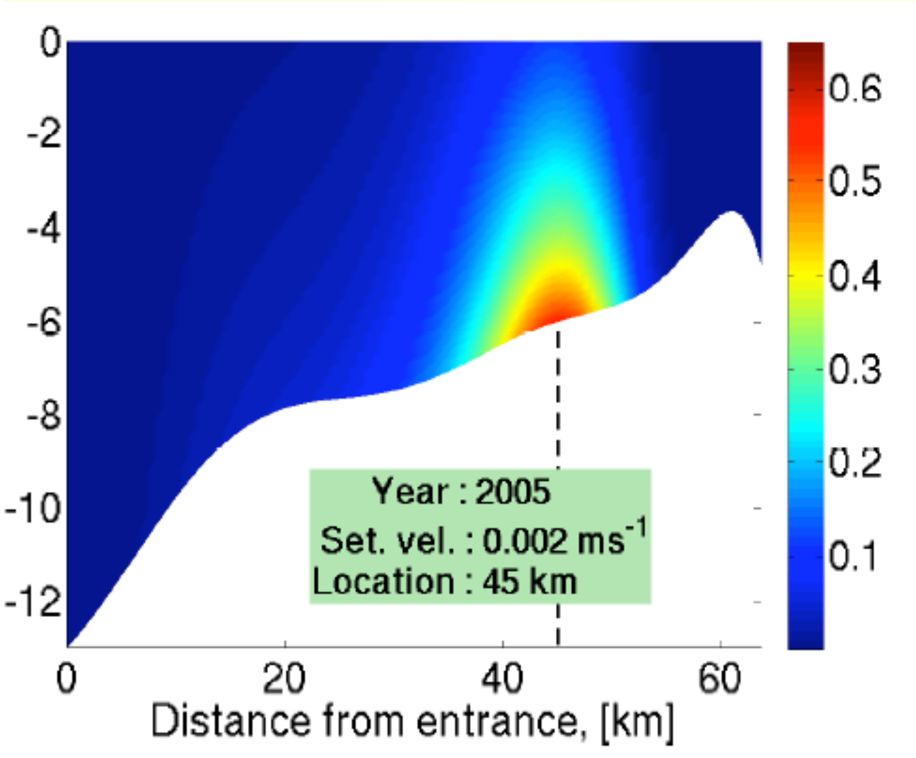



The behavior of the $T_{M2}$ component has changed due to changes in the externally prescribed $M_4$ :

**TIDAL ASYMMETRY MECHANISM**

# Model Results (9)

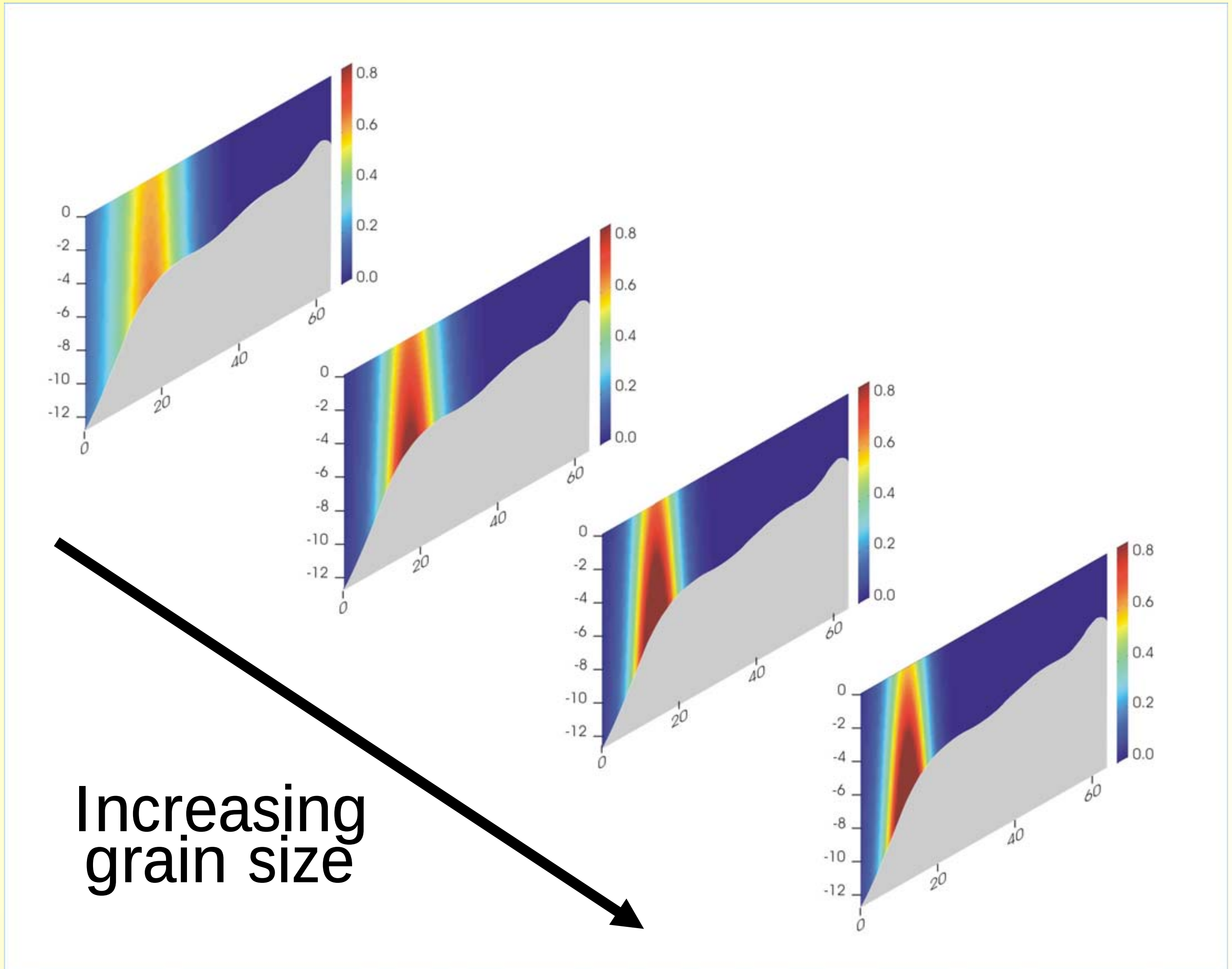



**CHANGE OF TRAPPING LOCATION DUE TO CHANGE IN TIDAL ASYMMETRY (CHARACTER OF M4 TIDAL WAVE)**

# Model Results (10)

## Dependency on grain size (1980)



# Model Results (10)

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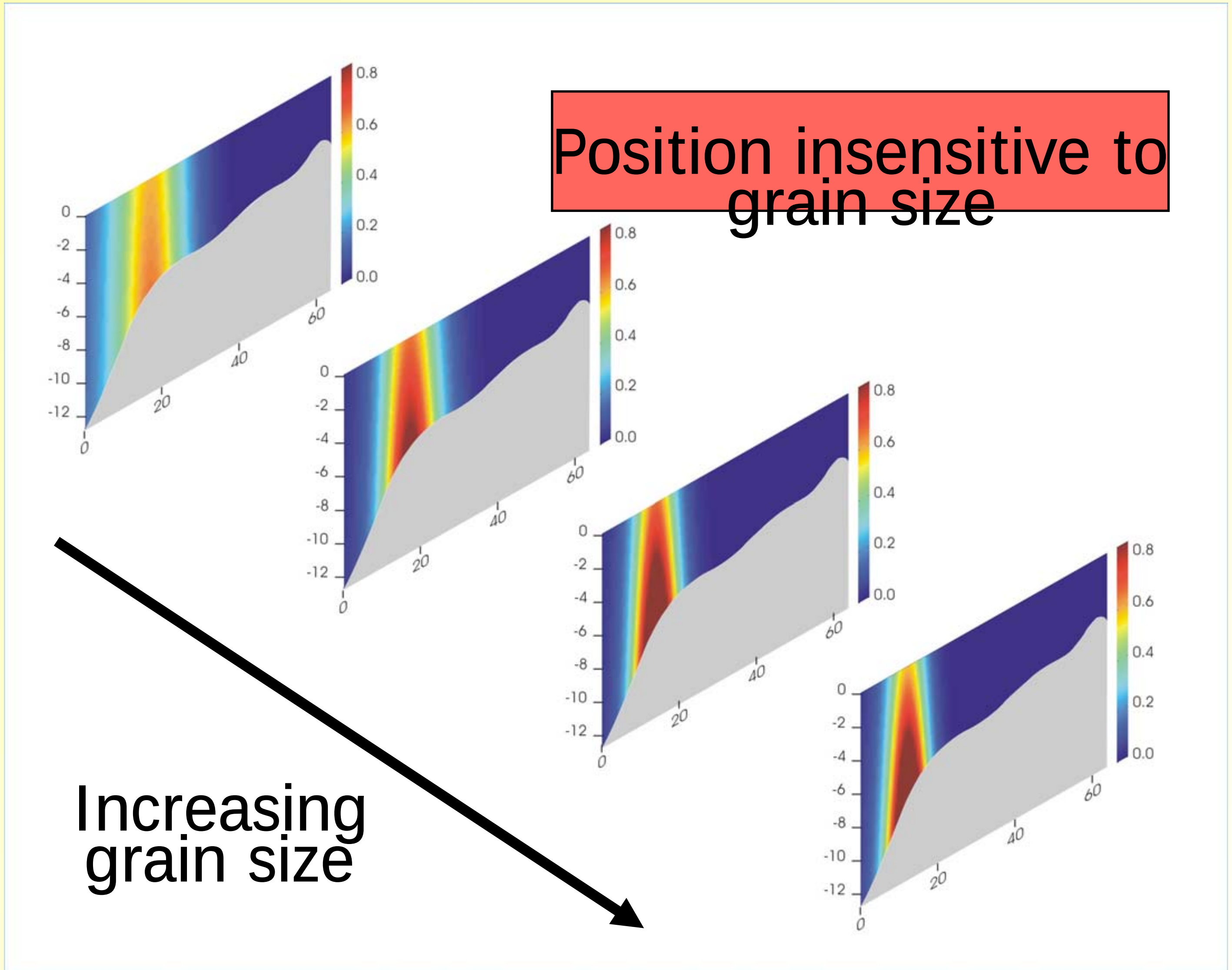
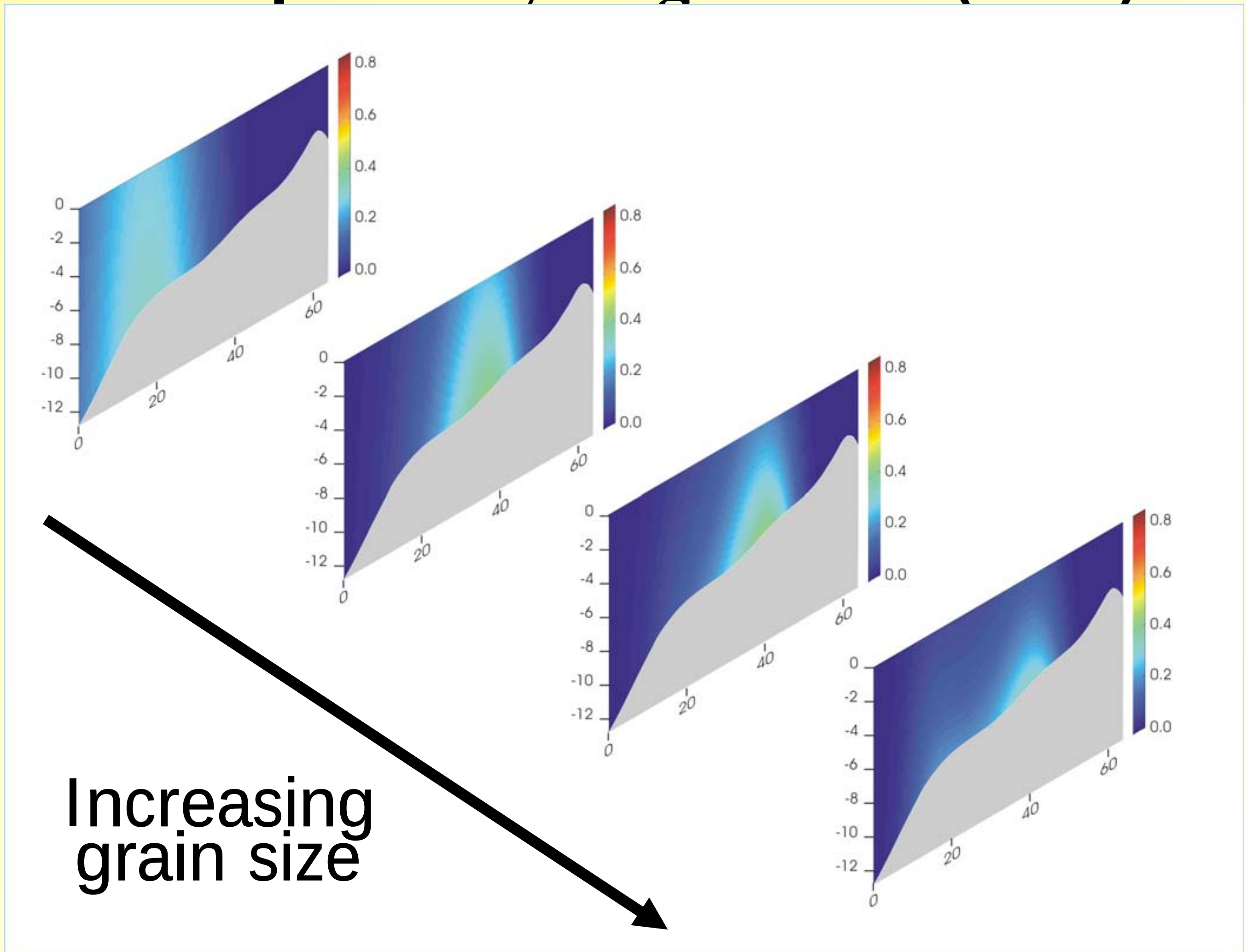


# Model Results (11)

## Dependency on grain size (2005)



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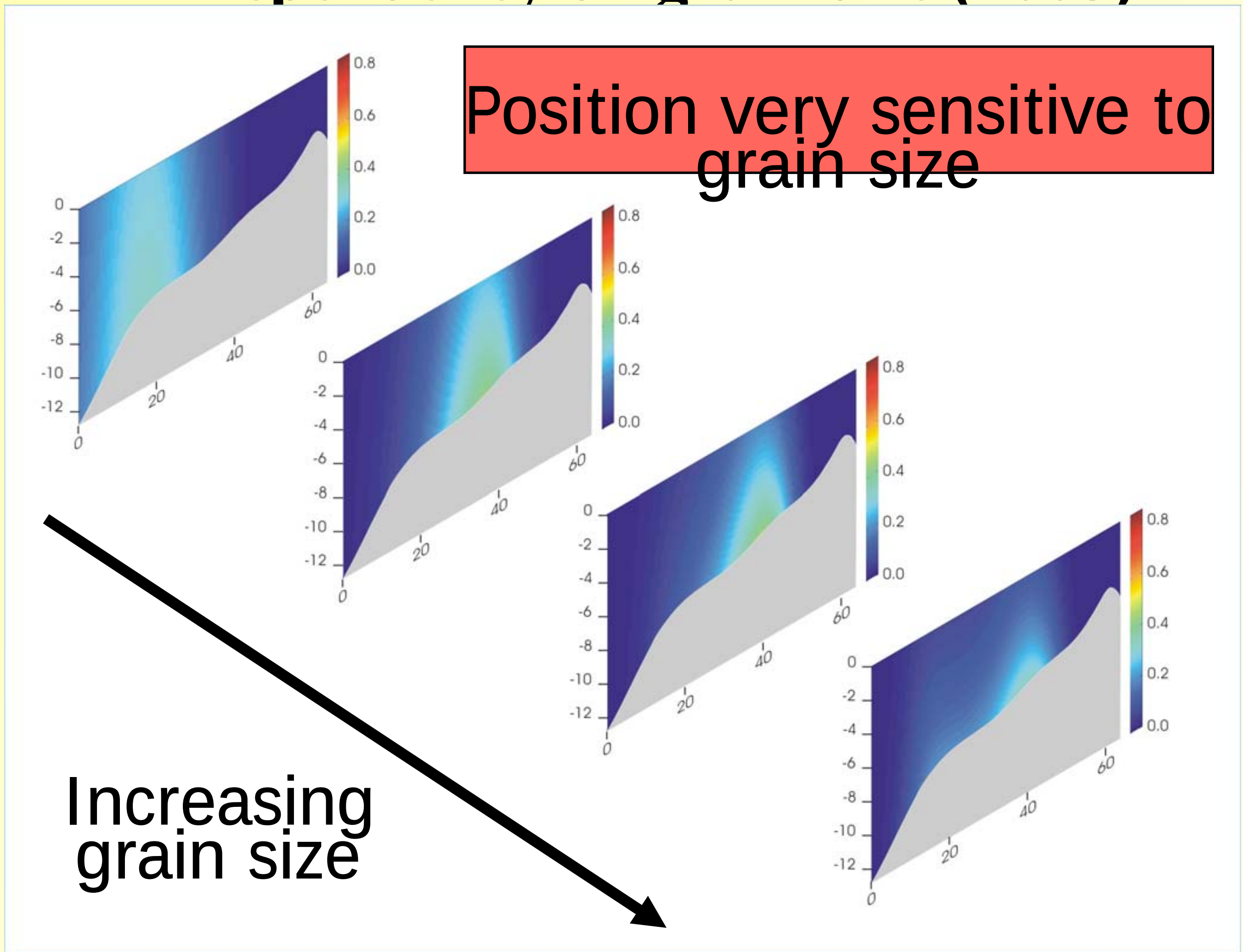
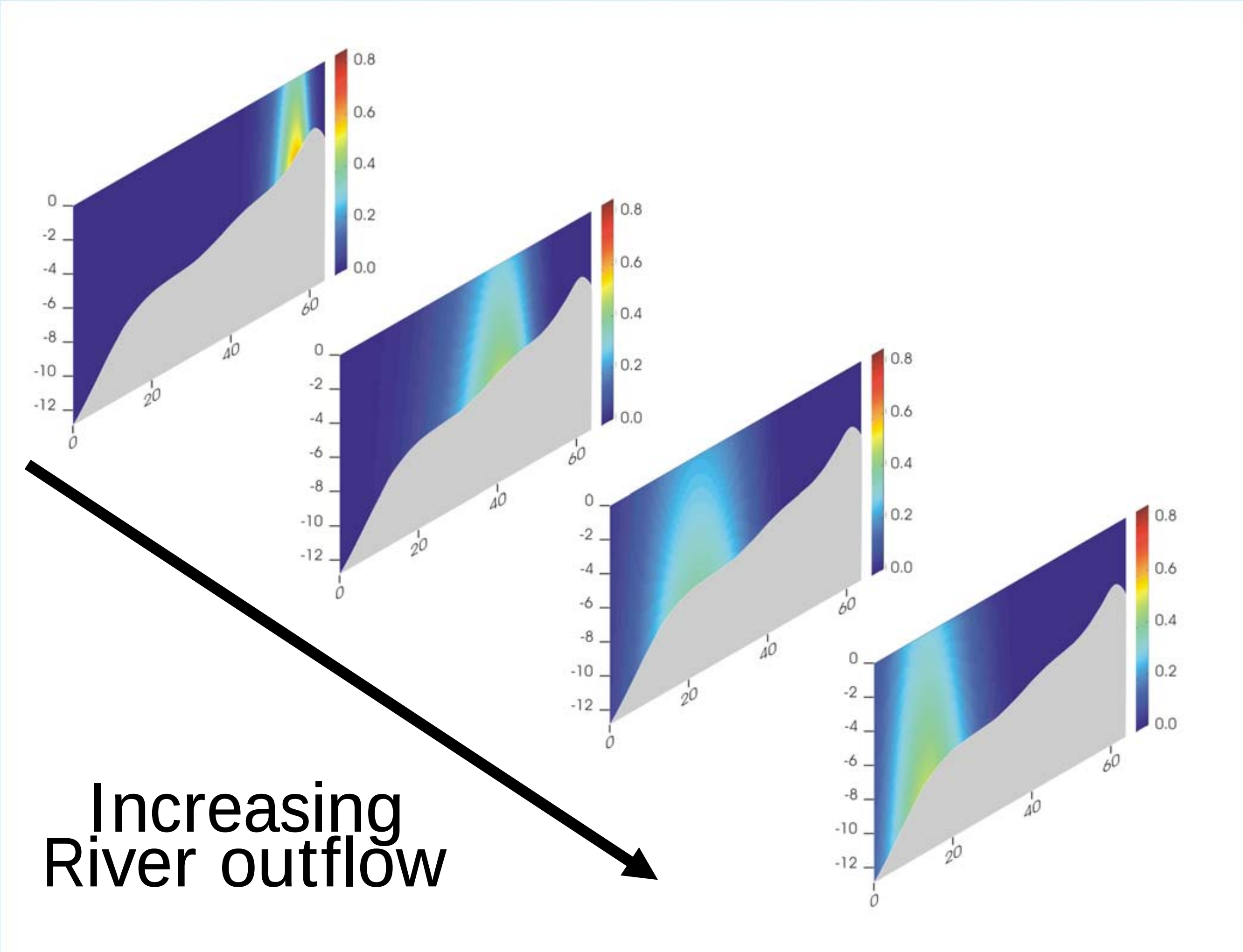


# Model Results (12)

## Dependency on river outflow (2005)



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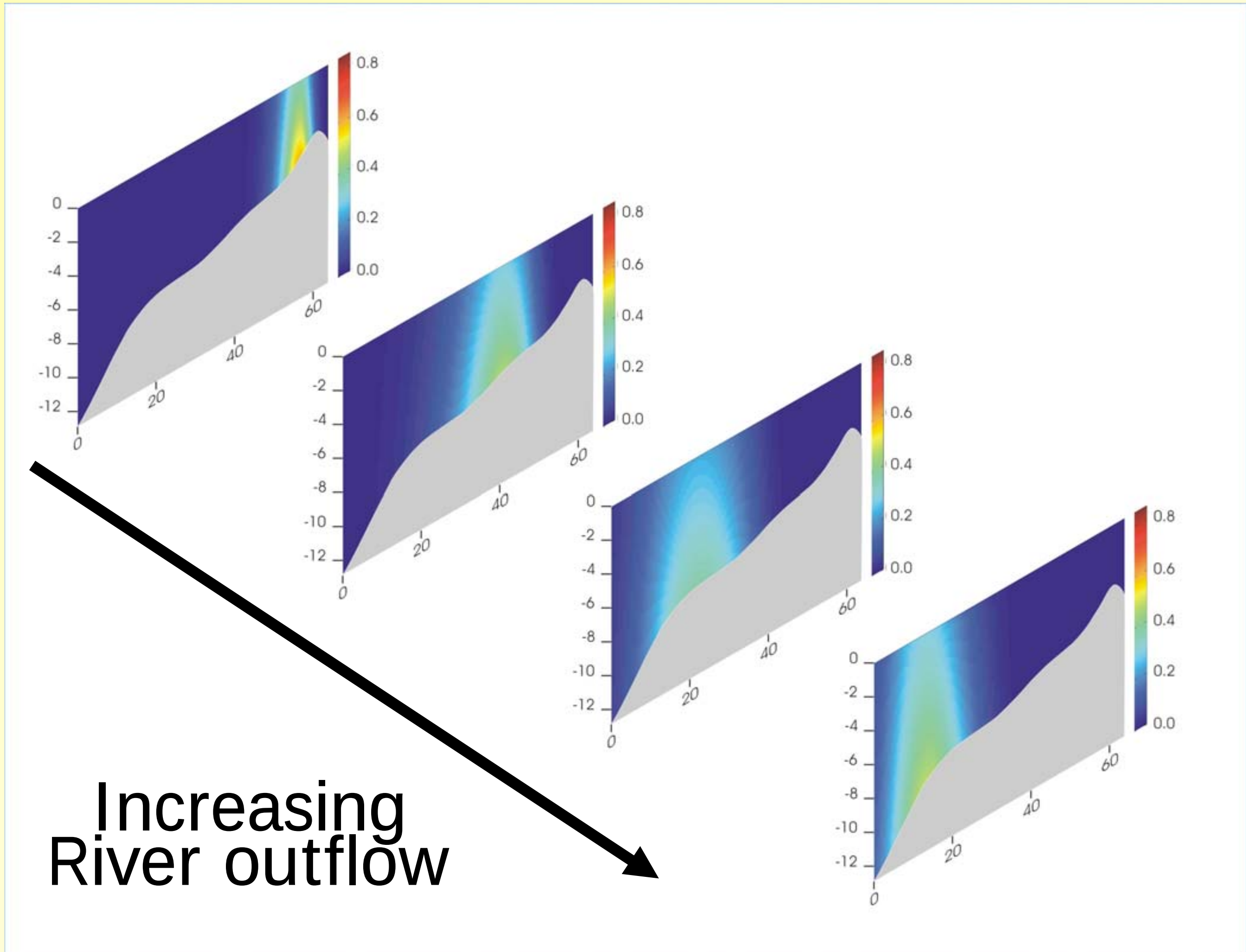


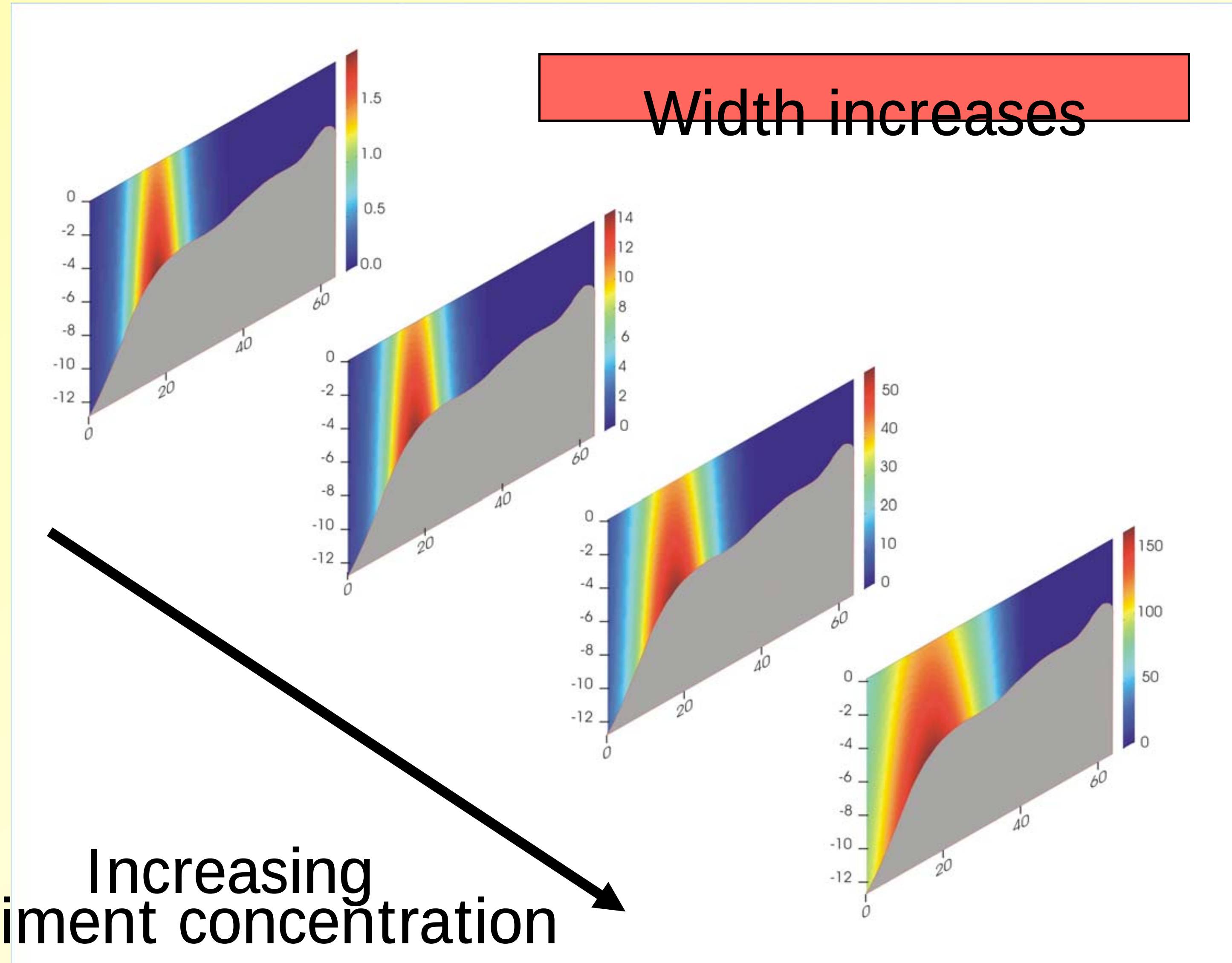
# Model Results (14)

**Extending the model by making density depend on Sediment concentration as well**

Results in a nonlinear differential equation for the sediment availability a(x)

# Model Results (15)

## **Results for the 1980 case**

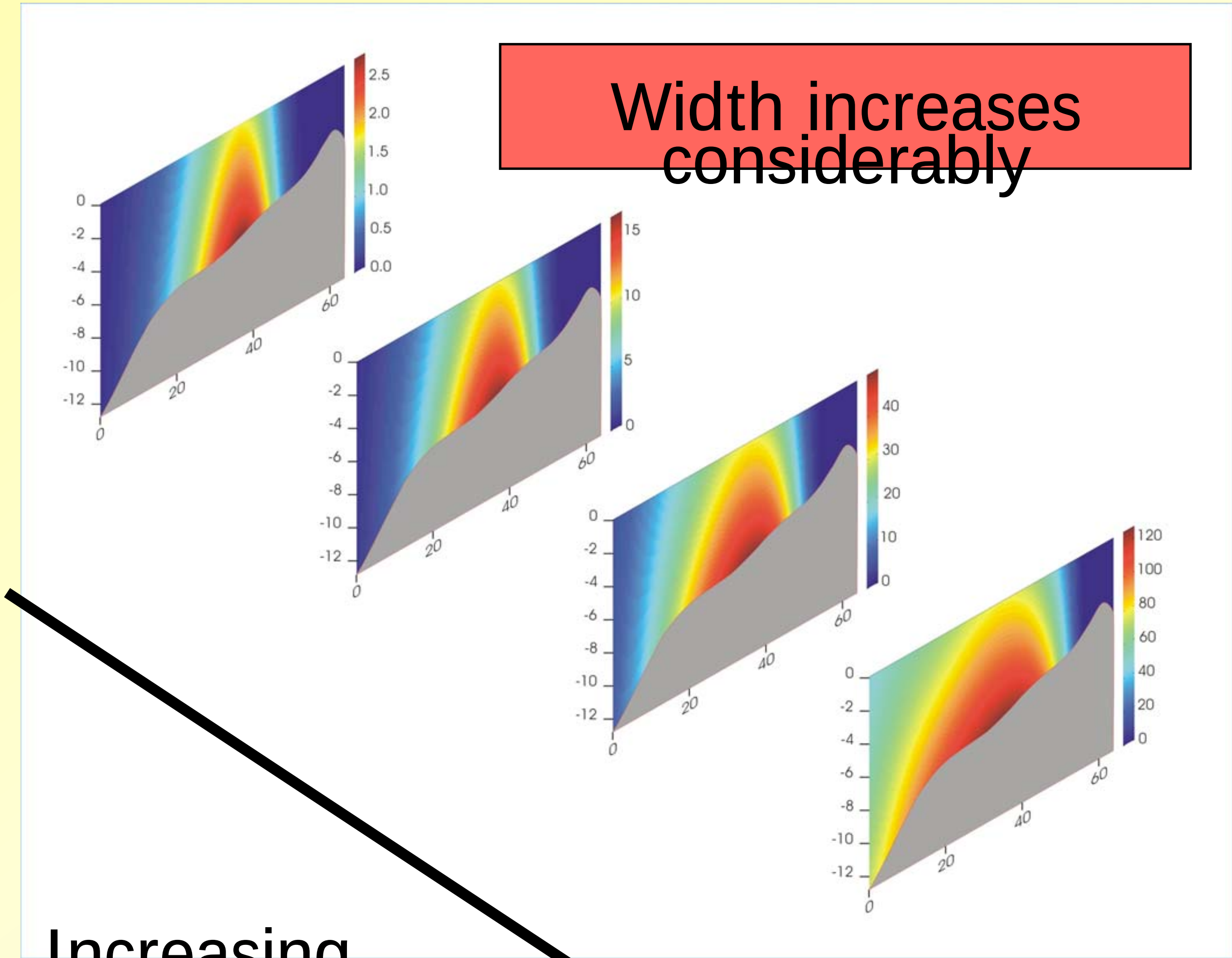


Width increases

Increasing
Sediment concentration

# Model Results (16)

## Results for the 2005 case




Width increases considerably

Increasing
Sediment concentration

# Conclusions

- Formation of ETMs can be modelled with an idealised model. Essential Ingredients:
  - Along-estuary varying erosion coefficient (~ layer of fine sediment)
  - Tidal asymmetry
- Physical mechanisms resulting in the ETMs can be understood using the idealised model.
- Difference in trapping of sediment in Ems Estuary mainly a result of the changes in tidal asymmetry between 1980 and 2005.
- Model sensitivities to parameters and parameterizations can be easily investigated. The trapping locations in 1980 are not very sensitive to parameter changes, in 2005 the locations change dramatically when changing parameters.