ASYMPTOTICS OF MOMENTS OF LINEAR RANDOM RECURRENCES

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A *linear random recurrence* is a sequence of random variables $\{X_n, n \in \mathbb{N}\}$ which satisfies the distributional equality

$$X_1 = 0, \qquad X_n \stackrel{d}{=} V_n + \sum_{r=1}^{K} A_r(n) X^{(r)}_{I^n_{(r)}}, \quad n \geq 2, \tag{1}$$

where, for every $r = 1, \ldots, K$, the sequence $\{X^{(r)}_k, k \in \mathbb{N}\}$ is a distributional copy of $\{X_k, k \in \mathbb{N}\}$, V_n is a random toll term, and $A_r(n) > 0$ is a random weight. It is assumed that $\{(I^n_{(1)}, \ldots, I^n_{(K)}, A_1(n), \ldots, A_K(n), V_n), n \geq 2\}$ and $\{X^{(1)}_n, n \in \mathbb{N}\}$,..., $\{X^{(K)}_n, n \in \mathbb{N}\}$ are independent.

The first step of asymptotic analysis of recurrences (1) is to find the asymptotics of moments $\mathbb{E}X^k_n$. This problem reduces to studying the recurrent equations of the form

$$a_1 = 0, \;\; a_n = b_n + \sum_{k=1}^{n-1} c_{nk} a_k, \;\; n \geq 2, \tag{2}$$

where $\{b_n, n \in \mathbb{N}\}$ and $\{c_{nk}, n \in \mathbb{N}, k < n\}$ are given numeric sequences. The purpose of the present talk is to propose a new method of obtaining the asymptotics of solutions to (2), as $n \to \infty$.

Our method is based on the technique of iterative functions and was initially developed in order to find the asymptotics of moments of the number of collisions Z_n and the absorption time T_n in the Poisson-Dirichlet coalescent. We have proved that both $\mathbb{E}Z^k_n$ and $\mathbb{E}T^k_n$, $k \in \mathbb{N}$, behave like the powers of the "log star" function which grows slower than any iteration of the logarithm.

References

[1] Marynych A. (2009). On the asymptotics of moments of linear random recurrences, (available at arXiv.org.).