Homework 2*

To be turned in next Monday!

Exercise 2-1 Prove the inequality $\Sigma(\mathcal{C}) \leq \sum_{n=1}^{\infty} \sum_{I \in \mathcal{C}_n} \mathrm{vol}(I)$ in the proof of Lemma 2.1.2 (see p. 21, line 8 from below). Note that "this is obvious" does not count as a correct answer (recall the care with which one has to prove the "obvious" Lemma 1.1.1).

Exercise 2-2 Also in the proof of Lemma 2.1.2 (see p. 22, lines 2,4 and 5) it is claimed that (i) $\Sigma(\mathcal{C}) = \Sigma(\mathcal{C}')$, (ii) $\mathcal{C}_i \supset \Gamma_i$ and (iii) $\mathcal{C}_1 \cap \mathcal{C}_2 = \emptyset$. Prove these three assertions.

Exercise 2-3 In Remark 2.1.3 one defines, given a closed set F, the sets G_n by $G_n := \{x \in \mathbb{R}^N : \exists_{y \in F} |x - y| < 1/n\}$. Prove (i) $G_n = \{x \in \mathbb{R}^N : \mathrm{dist}(x, F) < 1/n\}$, (ii) G_n is open. In part (i) we define $\mathrm{dist}(x, F) := \inf_{y \in F} |x - y|$, as usual.

Exercise 2-4 Prove that $|x + \Gamma|_e = |\Gamma|_e$ for every $x \in \mathbb{R}^N$ and every $\Gamma \subset \mathbb{R}^N$, as claimed in Remark 2.1.14.

Exercise 2-5 = Exercise 2.1.18

Exercise 2-6 = Exercise 2.1.20

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*Re: Measure and Integration, 14 January, 2002