Homework 3*

Starred exercises to be turned in next Thursday

Exercise 3-1 = Exercise 2.1.19

Exercise 3-2* Prove $\{y - x : x, y \in (q + A)\} \subset \{0\} \cup \mathbb{Q}^c$, as claimed in the last line of the proof of Theorem 2.1.17.

Exercise 3-3 Prove: $\mathcal{H} \subseteq \mathcal{P}(E)$ is a λ-system if and only if (a') $E \in \mathcal{H}$, (b') $\Gamma \in \mathcal{H} \Rightarrow \Gamma^c \in \mathcal{H}$, (c') $\{\Gamma_n\}_1^\infty \subseteq \mathcal{H} \Rightarrow \cup_1^\infty \Gamma_n \in \mathcal{H}$ if $\Gamma_n \cap \Gamma_m = \emptyset$ for $n \neq m$.

Exercise 3-4* The following claims are made in Example 3.1.5(ii): 1) The collection $\bar{\mathcal{B}}^\mu$ of all $\Gamma \subseteq E$ for which there exist $A, B \in \mathcal{B}$ such that $A \subseteq \Gamma \subseteq B$ and $\mu(B \backslash A) = 0$, is a σ-algebra. 2) Setting above $\bar{\mu}(\Gamma) := \mu(A)$ yields a map $\mu : \bar{\mathcal{B}}^\mu \to [0, \infty]$ that is a measure. Prove these two claims. *Hint:* Observe that the definition of $\bar{\mu}$ must also be inspected for being well-defined (i.e., non-ambiguous).

Exercise 3-5 = Exercise 3.1.8

Exercise 3-6 = Exercise 3.1.9

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*Re: Measure and Integration, 21 January, 2002