Homework 4*

Starred exercises to be turned in next Thursday

Exercise 4-1 = Exercise 3.1.9. Use this also to show that the original definition of the collection $\bar{\mathcal{B}}_{\mathbb{R}^N}$ of all Lebesgue measurable subsets can now be interpreted as follows: $\bar{\mathcal{B}}_{\mathbb{R}^N}$ is the completion of the Borel σ field $\mathcal{B}_{\mathbb{R}^N}$ with respect to the Lebesgue measure.

Exercise 4-2 Let E be a set and $\mathcal{B}$ a σ-field on E. Prove: $\mu : \mathcal{B} \to [0, \infty]$ is a measure if and only if (1) $\mu \not\equiv \infty$ (i.e. μ is not identically equal to ∞), (2) μ is σ-additive.

Exercise 4-3* Let $E := \mathbb{R}$ and let $\mathcal{C}$ be the collection of all singletons $\{x\}$, $x \in E$.

(a) Give a complete characterization of $\mathcal{A}(E, \mathcal{C})$.

(b) Give a complete characterization of $\mathcal{B} := \sigma(E, \mathcal{C})$.

(c) Let $\mu : \mathcal{B} \to [0, \infty]$ be a measure such that $\mu(\{x\}) = 1$ for every $x \in E$. Give a complete description of $\mu(B)$ for every $B \in \mathcal{B}$.

Exercise 4-4* Prove: $\mathcal{B}_{\mathbb{R}}$ is equal to $\sigma(\mathbb{R}, \mathcal{C})$, where $\mathcal{C}$ is the collection of all intervals of the form $(\alpha, \beta]$ with $\alpha, \beta \in \mathbb{R}$, $\alpha < \beta$.

Exercise 4-5 = Exercise 3.1.10

Exercise 4-6 = Exercise 3.1.12

*Re: Measure and Integration, 28 January, 2002