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Opgave 1 [30 pt.] a. Zij $f : \mathbb{R}^n \to \mathbb{R}$ een differentieerbare convexe functie en zij $S \subset \mathbb{R}^n$ nietleeg en convex. Beschouw het probleem om $f(\mathbf{x})$ te minimaliseren over alle $\mathbf{x} \in S$. Zij $\mathbf{x}_* \in S$. Bewijs de volgende FONSC voor optimaliteit: $\mathbf{x}_*$ is globaal minimum $\Leftrightarrow \nabla f(\mathbf{x}_*) \cdot (\mathbf{x} - \mathbf{x}_*) \geq 0$ voor elke $\mathbf{x} \in S$. *Hint:* Uit de handout "Intermezzo ..." weet je dat de volgende ongelijkheid geldt: $f(\mathbf{x}) \geq f(\mathbf{z}) + \nabla f(\mathbf{z}) \cdot (\mathbf{x} - \mathbf{z})$ voor elk paar $\mathbf{z}, \mathbf{x} \in \mathbb{R}^n$.
b. Beschouw nu voor vaste $\mathbf{y} \in \mathbb{R}^n$ het beste approximatie probleem uit hoofdstuk 2 van de syllabus: minimaliseer $\|\mathbf{s} - \mathbf{y}\|^2$ over alle $\mathbf{s} \in S$, waarbij S is als in onderdeel a. Ga na wat de specialisatie van de FONSC uit onderdeel a hier oplevert.
c. Beschouw tenslotte het beste approximatieprobleem uit onderdeel b voor de speciale situatie met $S := \{\lambda \mathbf{a} : \lambda \in \mathbb{R}_+\}$. Hier is $\mathbf{a} \in \mathbb{R}^n$ vast, $\mathbf{a} \neq \mathbf{0}$. Druk dan $\mathbf{x}_*$ uit in termen van $\mathbf{a}$. Illustreer je uitkomst m.b.v. een tweedimensionaal plaatje.

SOLUTION a. $\Leftarrow$: The inequality recalled in the hint gives for any $\mathbf{x} \in S$

$$f(\mathbf{x}) \geq f(\mathbf{x}_*) + \underbrace{\nabla f(\mathbf{x}_*) \cdot (\mathbf{x} - \mathbf{x}_*)}_{\geq 0} \geq f(\mathbf{x}_*),$$

so $\mathbf{x}_*$ is a global minimum.

$\Rightarrow$: **Method 1.** Fix any $\mathbf{x} \in S$ and define $\phi : [0,1] \to \mathbb{R}$ by $\phi(t) := f(t\mathbf{x} + (1-t)\mathbf{x}_*)$ (see for instance Proposition 1.2 of "Intermezzo..."). Now $\phi(0) = f(\mathbf{x}_*)$ is its minimum value over $[0,1]$, so because 0 is the left boundary point of the interval $[0,1]$ it follows that $\phi'(0) \geq 0$, where $\phi'(0) = \nabla f(\mathbf{x}_*) \cdot (\mathbf{x} - \mathbf{x}_*)$ holds by the chain rule for differentiation.

$\Rightarrow$: **Method 2.** Fix any $\mathbf{x} \in S$. Then for every $t \in]0,1]$ the point $t\mathbf{x} + (1-t)\mathbf{x}_*$ belongs to S, causing $f(\mathbf{x}_*) \leq f(\mathbf{x}_* + t(\mathbf{x} - \mathbf{x}_*))$ by the global minimum property of $\mathbf{x}_*$. Hence, the chain rule for differentiation gives

$$0 \leq \frac{f(\mathbf{x}_* + t(\mathbf{x} - \mathbf{x}_*)) - f(\mathbf{x}_*)}{t} \to \nabla f(\mathbf{x}_*) \cdot (\mathbf{x} - \mathbf{x}_*) \text{ for } t \downarrow 0.$$

b. In terms of part a, this problem corresponds to $f(\mathbf{x}) := \|\mathbf{x} - \mathbf{y}\|^2$. Here f is strictly convex, because its Hessian is $H_f = 2I$, (with I the $n \times n$-identity matrix), which is clearly positive definite. Observe also that $\nabla f(\mathbf{x}) = 2(\mathbf{x} - \mathbf{y})$. So part a gives the following: $\mathbf{x}_*$ is unique best approximant of $\mathbf{y}$ in $S \Leftrightarrow 2(\mathbf{x}_* - \mathbf{y}) \cdot (\mathbf{x} - \mathbf{x}_*) \geq 0$ for every $\mathbf{x} \in S$. The latter is cearly equivalent to the obtuse angle property of Theorem 2.3.6 in the main syllabus.

c. **Note:** this is *exactly* Example 2.3.13 in the main syllabus and it was *exactly* homework problem 2 on 16-2-11! Let $\mathbf{x}_* \in K$ be best approximant of $\mathbf{y}$. By definition of K you know that $\mathbf{x}_* = \alpha_* \mathbf{a}$ for some $\alpha_* \geq 0$.

Method 1: application of part b. According to part b, optimality of $\mathbf{x}_*$ is equivalent to $(\mathbf{y} - \mathbf{x}_*) \cdot (\mathbf{x} - \mathbf{x}_*) \leq 0$ for every $\mathbf{x} \in K$. By the simple nature of K, this is equivalent to

$$(\mathbf{y} - \mathbf{x}_*) \cdot (\alpha \mathbf{a} - \mathbf{x}_*) \leq 0 \text{ for every } \alpha \geq 0. \tag{1}$$

Taking $\alpha = 0$ in (1) gives $(\mathbf{y} - \mathbf{x}_*) \cdot \mathbf{x}_* \geq 0$. Also, letting α go to $+\infty$ in (1) implies $(\mathbf{y} - \mathbf{x}_*) \cdot \mathbf{a} \leq 0$, so you conclude that $(\mathbf{y} - \mathbf{x}_*) \cdot \mathbf{x}_* \leq 0$ holds as well. These two inequalities together give $(\mathbf{y} - \mathbf{x}_*) \cdot \mathbf{x}_* = (\mathbf{y} - \mathbf{x}_*) \cdot \alpha_* \mathbf{a} = 0$. Hence, you can distinguish the following two cases:

Case 1: $\alpha_* = 0$. In this case $\mathbf{x}_* = \mathbf{0}$, so (1) comes down to $\mathbf{y} \cdot \mathbf{a} \leq 0$.

Case 2: $\alpha_* > 0$. In this case $(\mathbf{y} - \mathbf{x}_*) \cdot \alpha_* \mathbf{a} = 0$ turns into $(\mathbf{y} - \alpha_* \mathbf{a}) \cdot \mathbf{a} = 0$, i.e., into $\alpha_* = \mathbf{y} \cdot \mathbf{a} / \|\mathbf{a}\|^2$.

Conclusion from cases 1-2: if the angle ϕ between $\mathbf{y}$ and $\mathbf{a}$ is obtuse (i.e., $\pi/2 \leq \phi \leq 3\pi/2$), then $\mathbf{0}$ is the best approximation in K of $\mathbf{y}$ and if the angle ϕ is sharp (i.e., $0 \leq \phi \leq \pi/2$ or $3\pi/2 \leq \phi \leq 2\pi$), then the best approximation in K of $\mathbf{y}$ is $\frac{\mathbf{y}\cdot\mathbf{a}}{\|\mathbf{a}\|^2}\mathbf{a}$, so in this case the best approximation coincides with the ordinary orthogonal projection of $\mathbf{y}$ on the entire linear space spanned by $\mathbf{a}$.

Method 2: elementary scalar optimization arguments. Let $\phi(\alpha) := \|\mathbf{y}-\alpha\mathbf{a}\|^2 = \alpha^2\|\mathbf{a}\|^2 - 2\alpha\mathbf{y}\cdot\mathbf{a} + \|\mathbf{y}\|^2$, $\alpha \geq 0$. Note that ϕ is just a convex parabolic function with derivative $\phi'(\alpha) = 2\alpha\|\mathbf{a}\|^2 - 2\alpha\mathbf{y}\cdot\mathbf{a}$. The optimality of $\mathbf{x}_* = \alpha_*\mathbf{a}$ means that $\phi(\alpha_*) \leq \phi(\alpha)$ for every $\alpha \in [0,+\infty[$.

Case 1: $\alpha_* = 0$. In this case $\phi'(\alpha_*) \geq 0$ holds as FONSC for the left boundary point $\alpha_* = 0$. This comes down to $\mathbf{y}\cdot\mathbf{a} \leq 0$.

Case 2: $\alpha_* > 0$. In this case α_* is interior to $[0,+\infty[$; hence $\phi'(\alpha_*) = 0$ holds as FONSC, which means $2\alpha_*\|\mathbf{a}\|^2 - 2\mathbf{y}\cdot\mathbf{a} = 0$, whence $\alpha_* = \mathbf{y}\cdot\mathbf{a}/\|\mathbf{a}\|^2$.

Opgave 2 [35 pt.] Beschouw een consument wiens preferenties kunnen worden weergegeven door de nutsfunctie $u(x_1,x_2) := x_1^\alpha(x_2+1)$ op $X := \mathbb{R}^2_+$. Hier is $\alpha > 0$ een parameter.
a. Beschouw het probleem ($\mathbb{U}_2$) Bepaal met behulp van een figuur de Marshalliaanse vraagbundel voor $p_1 = p_2 = y = 1$ en $\alpha = 2$.
b. Bepaal de Marshalliaanse vraagbundel(s) van deze consument. *Hint:* voor $x_1 > 0$ (!) vormt $\log(u(x_1,x_2))$ een concave functie op $\mathbb{R}_{++}\times\mathbb{R}_+ \subset X$.
c. Analyseer het limietgedrag van de bundels uit onderdeel b voor (i) de situatie waarbij $\alpha \to 0$ en (ii) de situatie waarbij $\alpha \to \infty$.
d. Bepaal eveneens de Hicksiaanse vraagbundel(s) van deze consument.
e. Verifieer de correctheid van je uitkomsten in onderdelen b en d voor $\alpha = 2$ door concreet de Slutsky decompositie te verifiëren voor de situatie $i = j = 1$.

Solution. a. A figure of this situation (omitted) clearly shows that the corner point $(1,0)$ is the Marshallian demand bundle, based on the fact that the slope of the budget line is -1 and the slope of the indifference curve passing through $(1,0)$ is -2.

b. The original UMP is

$$\text{maximize } x_1^\alpha(x_2+1) \text{ over all } x_1 \geq 0 \text{ and } x_2 \geq 0 \text{ with } p_1x_1 + p_2x_2 \leq y.$$

If one excludes the trivial case $y = 0$, then any bundle (x_1,x_2) with $x_1 = 0$ is inferior, because the utility function has its lowest value (zero) precisely for such bundles. Therefore the original UMP may be reformulated as

$$\text{maximize } x_1^\alpha(x_2+1) \text{ over all } x_1 > 0 \text{ and } x_2 \geq 0 \text{ with } p_1x_1 + p_2x_2 \leq y$$

and then also as

$$\text{maximize } \tilde{u}(x_1,x_2) := \alpha\log(x_1) + \log(x_2+1) \text{ over all } x_1 > 0 \text{ and } x_2 \geq 0 \text{ with } p_1x_1 + p_2x_2 \leq y \quad (2)$$

by using the strictly increasing transformation $t \mapsto \log(t)$ on $u(]0,+\infty[\times[0,+\infty[) =]0,+\infty[$. Note also that $\tilde{u}$ is strictly increasing on $X_0 :=]0,+\infty[\times[0,+\infty[$. Solving $\tilde{u}_{x_1}/p_1 = \tilde{u}_{x_2}/p_2$ yields $\alpha x_1^{-1}/p_1 = (x_2+1)^{-1}/p_2$, whence $x_2 = \frac{p_1x_1}{\alpha p_2} - 1$. When substituted in the budget equation, this gives $x_1 = \bar{x}_1 := \alpha(y+p_2)/((\alpha+1)p_1)$ and then also $x_2 = \bar{x}_2 := \frac{p_1\bar{x}_2}{\alpha p_2} - 1 = (y-\alpha p_2)/((\alpha+1)p_2)$. But this solution only counts if $\bar{x}_2 \geq 0$, i.e., if $y \geq \alpha p_2$; otherwise, $\tilde{u}_{x_1}/p_1 = \tilde{u}_{x_2}/p_2$ has no solution and then the only viable corner solution is $(\frac{y}{p_1},0)$, because of what was said above. This leads to the following subdivision.

Case 1: $y \geq \alpha p_2$. Clearly, the function $\tilde{u}$ is strictly concave on X_0, because $t \mapsto \log(t)$ is strictly concave. Similar to Theorem 5.1 in "Intermezzo", it thus follows from the above that $\bar{\mathbf{x}}$ is the unique global maximum for (2). By the monotone transformation $u = \exp(\tilde{u})$ it follows immediately that $\bar{\mathbf{x}}$ is also the unique global maximum for the original UMP.

Case 2: $y < \alpha p_2$. As mentioned above, in this case the global maximum must be $(\frac{y}{p_1},0)$.

Conclusion from cases 1-2: the Marshallian demand bundle is as follows:

$$\mathbf{x}^*(p_1,p_2,y;\alpha) := \begin{cases} (\frac{\alpha(y+p_2)}{(\alpha+1)p_2}, \frac{y-\alpha p_2}{(\alpha+1)p_2}) & \text{if } y \geq \alpha p_2, \\ (\frac{y}{p_1},0) & \text{if } y < \alpha p_2. \end{cases}$$

Remark 1: Observe how the outcome of part a – which deals with a very special case and was solved by another method – can be used as a check for correctness of your outcome in part b.

Remark 2: Ignoring the hint and using the UMP solution method instead leads to some complications: in step 1 one obtains the above $\bar{\mathbf{x}}$, but then in step 2 of that method a comparison must be made, for $y \geq \alpha p_2$, between the values $u(\bar{\mathbf{x}}) = (y+p_2)^{\alpha+1}\alpha^\alpha/((\alpha+1)^{\alpha+1}p_2p_1^\alpha)$ and $u(y/p_1, 0) = (y/p_1)^\alpha$. This requires the use of monotonicity of $h(y) := (y+p_2)^{\alpha+1}\alpha^\alpha/((\alpha+1)^{\alpha+1}p_2p_1^\alpha) - (y/p_1)^\alpha$, rather similar to what was done in Example 3.4.9 of the main syllabus.[1]

c. (i) Clearly, the second "fork" in the previous expression cannot be maintained if $\alpha \to 0$, so you easily get $\lim_{\alpha\to 0} \mathbf{x}^*(p_1, p_2, y; \alpha) = (0, y/p_2)$. This is not as strange as it seems at first sight (considering the elimination of points $x_1 = 0$ performed in part b), because for $\alpha = 0$ (i.e., "in the limit") the utility function is $u(x_1, x_2) = x_2 + 1$, which is indeed maximized by setting $x_1 = 0$ and $x_2 = y/p_2$.

(ii) This time the first "fork" cannot be maintained as $\alpha \to \infty$, so you trivially find $\lim_{\alpha\to\infty} \mathbf{x}^*(p_1, p_2, y; \alpha) = (y/p_1, 0)$.

d. Fix $v \in u(X) = [0, +\infty[$. As is known from the syllabus, you may take $v > u(0, 0) = 0$ w.l.o.g. This causes any solution to the EMP to belong to the above set X_0. So the original EMP is equivalent to

$$\text{minimize } p_1x_1 + p_2x_2 \text{ over all } (x_1, x_2) \in X_0 \text{ such that } \tilde{u}(x_1, x_2) \geq \log(v).$$

By part b you must now combine $\alpha x_1^{-1}/p_1 = (x_2+1)^{-1}/p_2$ (which again comes from $\tilde{u}_{x_1}/p_1 = \tilde{u}_{x_2}/p_2$) with $\alpha \log(x_1) + \log(x_2 + 1) = \log(v)$. By standard calculations this gives $x_1 = \tilde{x}_1 := (\alpha v p_2/p_1)^\beta$ and $x_2 = \tilde{x}_2 := (\alpha p_2/p_1)^{-\alpha\beta}v^\beta - 1$, where $\beta := 1/(1+\alpha)$. However, this solution is only acceptable if $\tilde{x}_2 \geq 0$, i.e., if $v \geq (\alpha p_2/p_1)^\alpha$. So you make the following division:

Case 1: $v \geq (\alpha p_2/p_1)^\alpha$. In this case $\tilde{\mathbf{x}}$ is the unique Hicksian demand bundle, because of the strict concavity of $\tilde{u}$ on X_0 (observe that $\tilde{u}$ is continuous, so any Hicksian bundle must be efficient).

Case 2: $v < (\alpha p_2/p_1)^\alpha$. In this case the only optimality candidate is the single corner point of the set $\{(x_1, x_2) \in X_0 : \tilde{u}(x_1, x_2) \geq \log(v)\}$, which is obtained by setting $x_2 = 0$ and gives $x_1 = v^{1/\alpha}$.

Conclusion from cases 1-2: the Hicksian demand bundle is given by

$$\mathbf{x}^h(p_1, p_2, v; \alpha) := \mathbf{x}_* = \begin{cases} ((\alpha v p_2/p_1)^\beta, (\alpha p_2/p_1)^{-\alpha\beta}v^\beta - 1) & \text{if } v \geq (\alpha p_2/p_1)^\alpha, \\ (v^{1/\alpha}, 0) & \text{if } v < (\alpha p_2/p_1)^\alpha. \end{cases}$$

e. **Note:** Below Slutsky's decomposition is given for general $\alpha > 0$ instead of the special choice $\alpha = 2$. For $i = j = 1$ this decomposition states

$$\frac{\partial x_1}{\partial p_1}(p_1, p_2, y) = \frac{\partial x_1^h}{\partial p_1}(p_1, p_2, v(p_1, p_2, y)) - x_1(p_1, p_2, y)\frac{\partial x_1}{\partial y}(p_1, p_2, y),$$

where the notation suppresses the parameter α, which remains fixed. To prepare for the verification of this identity, you must determine $I := \frac{\partial x_1}{\partial p_1}(p_1, p_2, y)$, $II := v(p_1, p_2, y)$, $III := \frac{\partial x_1^h}{\partial p_1}(p_1, p_2, v)$, $III' := \frac{\partial x_1^h}{\partial p_1}(p_1, p_2, v(p_1, p_2, y))$ and $IV := \frac{\partial x_1}{\partial y}(p_1, p_2, y)$.

Calculation of I, II, IV: From the above formula for Marshallian demand it follows directly that

$$I = \begin{cases} -\frac{\alpha(y+p_2)}{(\alpha+1)p_1^2} & \text{if } y \geq \alpha p_2, \\ -\frac{y}{p_1^2} & \text{if } y < \alpha p_2 \end{cases}$$

and that

$$II = \begin{cases} \frac{(y+p_2)^{\alpha+1}\alpha^\alpha}{(\alpha+1)^{\alpha+1}p_2p_1^\alpha} & \text{if } y \geq \alpha p_2, \\ (y/p_1)^\alpha & \text{if } y < \alpha p_2. \end{cases}$$

and

$$IV = \begin{cases} \frac{\alpha}{(\alpha+1)p_1} & \text{if } y \geq \alpha p_2, \\ \frac{1}{p_1} & \text{if } y < \alpha p_2 \end{cases}$$

[1]Briefly: by $h(\alpha p_2) = 0$ it is enough to prove $h'(y) > 0$ for $y > \alpha p_2$. Writing $\gamma := y/(\alpha p_2) > 1$, $h'(y) > 0$ comes down to $f(\alpha) > \gamma^{(\alpha-1)/\alpha}$, with $f(\alpha) := (\gamma\alpha + 1)/(\alpha + 1)$, which is obviously true, as $\min_{\alpha\geq 0} f(\alpha) = f(0) = \gamma$.

Calculation of III, III': From the above expression for Hicksian demand it follows directly that

$$III = \begin{cases} -\beta(\alpha v p_2)^\beta p_1^{-\beta-1} & \text{if } v \geq (\alpha p_2/p_1)^\alpha, \\ 0 & \text{if } v < (\alpha p_2/p_1)^\alpha. \end{cases}$$

Finally, to determine III', you must check that the upper "fork" in the expression for Hicksian demand, with v replaced by $v(p_1, p_2, y)$ as in the upper "fork" of II, conforms to its defining condition $y \geq \alpha p_2$. Indeed, one has

$$\frac{(y+p_2)^{\alpha+1}\alpha^\alpha}{(\alpha+1)^{\alpha+1}p_2 p_1^\alpha} \geq (\alpha p_2/p_1)^\alpha \Leftrightarrow \frac{(y+p_2)^{\alpha+1}}{(\alpha+1)^{\alpha+1}} \geq p_2^{\alpha+1} \leftrightarrow y \geq \alpha p_2.$$

Hence

$$III' = \begin{cases} -\beta(\alpha[\frac{(y+p_2)^{\alpha+1}\alpha^\alpha}{(\alpha+1)^{\alpha+1}p_2 p_1^\alpha}]p_2)^\beta p_1^{-\beta-1} & \text{if } y \geq \alpha p_2, \\ 0 & \text{if } y < \alpha p_2 \end{cases}$$

An easy calculation then shows that the expression in the upper fork above equals $-\alpha(y+p_2)/(\alpha+1)p_2)^2$, which means that one must now verify for $y \geq \alpha p_2$

$$-\frac{\alpha(y+p_2)}{(\alpha+1)p_1^2} \stackrel{?}{=} -\frac{\alpha}{(\alpha+1)^2 p_1^2} - \frac{\alpha^2(y+p_2)}{(\alpha+1)^2 p_1^2},$$

which is evidently true, and for $y < \alpha p_2$ the verification is trivial: $-y/p_1^2 = 0 - (y/p_1)(1/p_1)$.

Opgave 3 [30 pt.] Een markt voor een bepaald product omvat firma's die alle dezelfde kostenfunctie $C(q) = 6q^2 - 5q + 6$, $q \geq 0$, hebben. Zij kunnen de marktprijs $p > 0$ niet beïnvloeden, en handelen dus als "price takers". De totale marktvraag naar het product wordt gegeven door de functie $D(p) = 148 + \frac{64}{p}$.

a. Bepaal voor de individuele firma in deze markt de optimale outputhoeveelheid q^* als functie van de marktprijs p.

b. Stel dat op de *korte termijn* 144 firma's actief zijn op deze markt (alle met bovenstaande kostenfunctie $C(q)$). Bepaal dan de korte termijn evenwichts-marktprijs p^* voor het product.

c. Beschouw nu dezelfde markt, maar op de *lange termijn*. Wat is dan (1) de evenwichtprijs p^{**} op deze markt? (2) het aantal firma's op deze markt?

d. De regering is niet tevreden met de evenwichtsprijs p^{**} in onderdeel c. Om deze te veranderen legt zij aan elke firma die op de markt van het product actief is, een lump sum heffing H op. Hoe groot moet H zijn opdat $p^{**} = 10$?

SOLUTION. a. The strictly concave function $\psi(q) := pq - C(q) = pq - 6q^2 + 5q - 6$ has $q^*(p) = (p+5)/12 > 0$ as its unique global maximum over $[0, +\infty[$ (by $\psi'(q^*(p)) = 0$).

b. Total short run supply is $S(p) := 144q^*(p) = 12(p+5)$. Solving $S(p) = D(p)$ easily gives $3p = 22 + \frac{16}{p}$, which is equivalent to $3p^2 - 22p - 16 = 0$. This implies $p = p^* := 8$ for the short run equilibrium price.

c. The long run equilibrium price is $p^{**} = p_{crit} := \min_{q>0} C(q)/q$ (indeed $p < p_{crit}$ implies that net profit $pq - C(q)$ is strictly negative at any output level $q \geq 0$, causing firms to leave the market, while $p > p_{crit}$ implies that net profit can be made strictly positive for some suitable output choice, causing firms to enter the market). The present case gives $p_{crit} = \min_{q>0} \phi(q) := 6q - 5 + 6q^{-1}$. It follows that $p^{**} = p_{crit} = \phi(1) = 7$ (set $\phi'(q) = 0$ and note that ϕ is strictly convex on $]0, +\infty[$), so $q^* := 1$ is the unique output choice which makes maximum profit under $p = 7$ equal to zero. At the equilibrium price $p^{**} = 7$ total demand is $D(7) = 148 + \frac{64}{7} = 1100/7$, so the long run equilibrium number of firms is $N^* = D(7)/q^* = 1100/7(\approx 157)$.

d. The extra lump sum taxation leads to the new cost function $C_{new}(q) = 6q^2 - 5q + 6 + H$ for each firm. The new long run critical price $p_{crit}^{new} = \min_{q>0} \phi_{new}(q) := 6q - 5 + (6+H)q^{-1}$ can be determined by $0 = \phi'_{new}(q) = 6 - (6+H)q^{-2}$, i.e., by $q = \sqrt{1 + \frac{H}{6}}$. This gives $p_{crit}^{new} = 12\sqrt{1 + \frac{H}{6}} - 5$, which the government wants to equal 10. Therefore, H follows from $\sqrt{1 + \frac{H}{6}} = 5/4$, which gives $H = 27/8$.