

Hg: 24 $X \sim \text{Bin}(n, p)$

a) Generalized LR-test von $H_0: p=0.5$ vs $H_A: p \neq 0.5$

$$\Lambda = \frac{\max_{p=0.5} L(p)}{\max_{p \in \{0.1\}} L(p)} = \frac{\max_{p=0.5} \binom{n}{x} p^x (1-p)^{n-x}}{\max_{p \in \{0.1\}} \binom{n}{x} p^x (1-p)^{n-x}} = \frac{\frac{1}{2}^n}{\binom{n}{x} \left(\frac{x}{n}\right)^x \left(\frac{n-x}{n}\right)^{n-x}}$$

$$= \frac{\binom{n}{x}}{x^x (n-x)^{n-x}}$$

b) Verwerp als Λ klein $\Rightarrow x^x (n-x)^{n-x}$ groot.

symmetrisch in $\frac{n}{2} \pm y$

$$\left(\frac{\frac{n}{2}+y}{n-(\frac{n}{2}+y)}\right)^{\frac{n}{2}+y} = \left(\frac{\frac{n}{2}+y}{\frac{n}{2}-y}\right)^{\frac{n}{2}+y}$$

$$\left(\frac{\frac{n}{2}-y}{n-(\frac{n}{2}-y)}\right)^{\frac{n}{2}-y} = \left(\frac{\frac{n}{2}-y}{\frac{n}{2}+y}\right)^{\frac{n}{2}-y}$$

} identiek

$\min_{x \in \{0, \dots, n\}}$ in $x = \frac{n}{2}$, verwerp als $|X - \frac{n}{2}|$ groot.

c) $R = \{x \mid |x - \frac{n}{2}| > R\} \Rightarrow -x + \frac{n}{2} > R \vee x - \frac{n}{2} > R$

$P(R) = 2 \cdot \sum_{i=0}^{\frac{n}{2}-R} P(i)$

$x > \frac{n}{2} + R$
 $x - \frac{n}{2} < -R$
 $x < -R + \frac{n}{2}$

1) $2(P(0) + P(1) + P(2)) = 2 \cdot \left(\frac{1}{2}^{10} + 10 \cdot \frac{1}{2}^{10} + 45 \cdot \frac{1}{2}^{10}\right) = \frac{7}{64} \sim 0.11$

2) $n=100, R=10$

$$P(|X - \frac{n}{2}| > R) = 2 P(X - \frac{n}{2} > R) = 2 P\left(\frac{X - E(X)}{\sigma} > \frac{R}{\sigma}\right) = 2 P\left(\frac{X - E(X)}{\sqrt{n \cdot \frac{1}{2} \cdot \frac{1}{2}}} > \frac{R}{\sqrt{n \cdot \frac{1}{2} \cdot \frac{1}{2}}}\right)$$

continuiteit's correctie

$$= 2 P\left(\frac{X - E(X)}{\sigma} > \frac{R + 0.5}{\sigma}\right) = 2 \cdot (1 - \Phi(z)) = 2 \cdot (1 - \Phi(2)) = 2 \cdot 0.054 = 0.108$$