

26 januari 2004

$$Q = X + Y$$
$$(Q, Z)^T = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix}. \text{ De covariantiematrix voor } (Q, Z)^T \text{ is } \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \Sigma \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ -1 & 5 \end{pmatrix}$$

Opgave 1 $E(\bar{Z}_5) = E(Z) = E(X) - E(Y) = 1 + 1 = 2$

$$\text{Var}(\bar{Z}_5) = \frac{1}{25} \sum_{i=1}^5 \text{Var}(Z_i) = \frac{1}{25} 5 \text{Var}(Z) = 1$$

$$\text{Cov}(Z, Q) = -1$$

$$E(S_z^2) = \text{Var}(Z) = 5$$

$$\text{Var}(\sqrt{2}S_z^2) = 2\text{Var}(S_z^2) = 2 \cdot 2(\sigma_z)^4/4 = 25 \text{ (opgave 9 H6, of hieronder)}$$

$$P(\bar{Z}_5 \leq S_z \frac{0.569}{\sqrt{5}} + 2) = P\left(\frac{\bar{Z}_5 - E(\bar{Z}_5)}{S_z/\sqrt{5}} \leq 0.569\right) = P(t_4 < 0.569) = 0.7$$

De verdeling van (Z, Q) is multivariaat normaal verdeeld met $\mu = (2, 0)^T$ en $\Sigma =$

$$\begin{pmatrix} 1 & -1 \\ -1 & 5 \end{pmatrix}$$

$$\text{Var}\left(\frac{(n-1)S^2}{\sigma^2}\right) := \text{Var}(\chi_{n-1}^2) \Rightarrow \frac{(n-1)^2}{\sigma^4} \text{Var}(S^2) = 2(n-1) \Rightarrow \text{Var}(S^2) = \frac{2\sigma^4}{n-1}.$$