Differentiable manifolds – hand-in sheet 2

Hand in by 2/Nov

Prelude to the exercise

Definition 1. A *Lie group* is a manifold endowed with a group structure such that multiplication and inversion are smooth maps, i.e.

$$G \times G \longrightarrow G \qquad (g,h) \mapsto g \cdot h$$
$$G \longrightarrow G \qquad g \mapsto g^{-1}$$

are smooth.

Exercise

1) Show that $O(n)$, the group of $n \times n$ orthogonal matrices, is a Lie group. Compute the tangent space of $O(n)$ at the identity element.

Hint: We have already seen that $GL(n, \mathbb{R})$ is a Lie group.