Differentiable manifolds – homework 4

Definition: A *Lie group* is a manifold G endowed with a group structure for which multiplication and inversion are smooth maps.

1) Let M and N be manifolds and $M \times N$ be their product. Show that $M \times N$ is a manifold. Let $m \in M$ and $n \in N$. Show that the maps

$$i_n : M \longrightarrow M \times N \qquad i_n(x) = (x, n);$$

$$i_m : N \longrightarrow M \times N \qquad i_m(y) = (m, y);$$

$$\pi_M : M \times N \longrightarrow M \qquad \pi(x, y) = x;$$

$$\pi_N : M \times N \longrightarrow N \qquad \pi(x, y) = y.$$

are smooth.

2) Let G be a Lie group and $g \in G$, show that the map $l_g : G \longrightarrow G$ given by $l_g(x) = g \cdot x$ is a diffeomorphism of G.

3) Show that $\mathbb{C} \backslash \{0\}$ with complex multiplication is a Lie group. Show that S^1, the set of complex numbers of norm 1 is also a Lie group.

4) Show that $\mathbb{H} \backslash \{0\}$ with quaternionic multiplication is a Lie group.

5) Show that the general linear group of $n \times n$ invertible real matrices, $Gl(n; \mathbb{R})$, is a Lie group.