Differentiable manifolds – homework 8

Exercises from the book: All exercises from Chapter 1.

1) Let M be a compact manifold and $\varphi : M \longrightarrow N$ be an injective immersion. Show that φ is an embedding.

2) Let $f : \mathbb{R}^2 \longrightarrow \mathbb{R}$ be given by

$$f(x_1, x_2) = x^2 - y^2$$

Determine the critical points and critical values of f.

3) Given a smooth map $\varphi : M \longrightarrow N$ it induces *pullback* maps on 0- and 1-forms , all of them denoted by φ^*, defined by

$$\begin{aligned} &\varphi^* : \Omega^0(N) \longrightarrow \Omega^0(M) && \varphi^* f = f \circ \varphi; \\ &\varphi^* : \Omega^1(N) \longrightarrow \Omega^1(M) && \alpha \mapsto \varphi^*\alpha; \\ & && \varphi^*\alpha|_p(X) = \alpha|_{\varphi(p)}(\varphi_* X) \qquad \forall X \in T_pM. \end{aligned}$$

Show that if $f \in \Omega^0(N)$, then $\varphi^* df = d(\varphi^* f)$.

4) Consider the following vector field defined in the manifold $\mathbb{C}^n$:

$$X(z_1, \cdots, z_n) = (iz_1, \cdots, iz_n),$$

or, in real coordinates,

$$X(x_1, y_1, \cdots, x_n, y_n) = (-y_1, x_1, \cdots, -y_n, x_n).$$

Compute the flow of X. Show that the flow of X preserves the sphere of radius r centered at the origin.