Sheet 4

Solve exercises 7 and 8 from chapter 1.1.

Exercise 1. Show that if $p : \tilde{X} \longrightarrow X$ is a covering space of X then the following homotopy lifting property holds:

c) If $F : Y \times I \longrightarrow X$ is continuous and $\tilde{F} : Y \times \{0\} \longrightarrow \tilde{X}$ is a lift of $F|_{Y\times\{0\}}$, i.e., the following diagram commutes

$$\begin{tikzcd} Y \times \{0\} \arrow[r, "\tilde{F}"] \arrow[d, "\mathrm{Id}"] & \tilde{X} \arrow[d, "p"] \\ Y \times \{0\} \arrow[r, "F|_{Y\times\{0\}}"] & X \end{tikzcd}$$

then there is a a unique extension of $\tilde{F}$ to $Y \times I$ for which the diagram below commutes

$$\begin{tikzcd} Y \times I \arrow[r, "\tilde{F}"] \arrow[d, "\mathrm{Id}"] & \tilde{X} \arrow[d, "p"] \\ Y \times I \arrow[r, "F"] & X \end{tikzcd}$$

Exercise 2. Let $p_1, \ldots, p_m$ be points in the n-dimensional sphere. Show that

$$S^n \backslash \{p_1, \ldots, p_m\} \simeq (m-1) \vee S^{n-1}.$$

Exercise 3. Let p be a point in the 2-torus T^2. Show that $T^2\backslash\{p\}$ deform retracts to $S^1 \vee S^1$. Conclude that $T^2\backslash\{p\} \simeq S^2\backslash\{p_1, p_2, p_3\}$.