Geometry and Topology – hand-in sheet 1

Hand in by 19/February

Exercise 1.

1. Consider the subspace $A \subset \mathbb{R}^2$ obtained by joining the points of in the set $\{(0,0), (-1,0), (-\frac{1}{2},0), \cdots, (-\frac{1}{n},0), \cdots\}$ to the point $(0,1)$ by a line segment. Show that A is contractible.

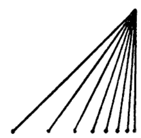

Figure 1: The space A.

2. Consider the subspace X of $\mathbb{R}^2$ obtained by joining the points of in the set $\{(0,0), (-1,0), (-\frac{1}{2},0), \cdots, (-\frac{1}{n},0), \cdots\}$ to the point $(0,1)$ by a line segment and the points of in the set $\{(0,0), (1,0), (\frac{1}{2},0), \cdots, (\frac{1}{n},0), \cdots\}$ to the point $(0,-1)$ by a line segment. Observe that $A \subset X$ and show that X/A is contractible.

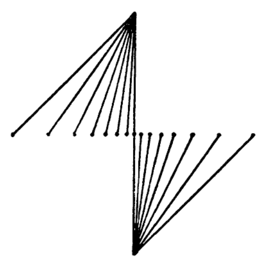

Figure 2: The space X.

3. Show that X is not contractible. You can follow the steps below or prove it in any other way you see fit.

   Step 1: Show that if $F : X \times I \longrightarrow X$ is a homotopy starting at the identity, then $F(0,0,t) = (0,0)$ for all t (use an open-closed argument on the interval). Conclude that if X is contractible, it deform retracts to $(0,0)$.

   Step 2: Show that if a space X deformation retracts to a point $x \in X$, then for each neighborhood U of x there exists a neighborhood $V \subset U$ containing x such that the inclusion $V \hookrightarrow U$ is null homotopic.

   Step 3: Show that, in the present example, $(0,0) \in X$ does not have this property.