Group theory – Sheet 2

The exercises from the book are 4.1, 4.3, 4.5, 4.10, 5.4, 5.5, 5.10, 5.12, 7.8.

1) Find all the subgroups of $\mathbb{Z}_n$.

2) Show that for m, n co-primes, $\mathbb{Z}_m \times \mathbb{Z}_n$ is isomorphic to $\mathbb{Z}_{mn}$. Is the result still true if m and n are not co-primes?

3) Let G be a group and $g \in G$. Consider the map $\varphi_g : G \longrightarrow G$ defined by $\varphi_g(h) = ghg^{-1}$. Show that φ is an isomorphism. Conclude that h and ghg^{-1} have the same order.

4) For $g, h \in G$, show that gh and hg have the same order.

5) Find all the elements of order 2 of D_n. (hint: consider separately the cases of n even and n odd).