

## Group theory – Sheet 3

The exercises from the book are 4.1, 4.4, 4.5, 4.6, 4.7, 3.4, 4.8, 4.9, 5.2, 5.5, 5.7, 5.11, 5.12.

1) Find all the subgroups of  $\mathbb{Z}_n$  (from last sheet).

2) Let  $n > 2$  and define a group by the following generators and relations:

$$G = \langle a, b : a^n = b^2 = e; bab^{-1} = a^k \rangle$$

Show that if  $k^2 \not\equiv 1 \pmod n$  then these relations imply that the order of  $a$  is less than  $n$ .

The point of this exercise is to show that writing a bunch of generators and relations may lead to a group with order smaller than one might expect at first glance.