

Registration form (basic details)

1a. Details of applicant

- Name, title(s): Marius Crainic
- Male/female: male
- Address for correspondence: Math. Inst., Utrecht Univ, PO Box 80.010, 3508 TA, Utrecht.

- Preference for correspondence in English: yes
- Telephone: (030) 2531429
- Fax: (030) 2518394
- E-mail: crainic@math.uu.nl
- Website: <http://www.math.uu.nl/people/crainic>

- Doctorate (date: dd/mm/yy): 03/04/2000
- Use of extension clause: no

1b. Title of research proposal:

Poisson topology

1c. Summary of research proposal

This is a mathematics proposal which belongs to the field of geometry. The main aim is to develop the field of "Poisson topology". We also aim at further understanding the fundamental work of Lie and Cartan on differential equations, with an eye towards applications. The notion of symmetry provides an overall tool for handling specific problems, interactions and applications. To give a non-specialised introduction into the background of the project, we explain the keywords.

"Geometry": geometry has its roots in the study of objects one finds in nature- they are either immediately intuitive (e.g. triangles, spheres) or are revealed by physical theories. Every field in geometry is characterized by certain aspects/properties that it studies. E.g., Riemannian geometry studies properties related to "lengths" and "angles". Poisson geometry, a central theme of the project, concentrates on the mathematical structure behind the equations of classical mechanics or quantization schemes.

"Poisson Topology": the topological study of geometrical objects refers to those properties which depend only on the notion of "continuity"- a transformation is called continuous if it transforms nearby points to nearby points. For instance, a circle and an ellipsis, although geometrically distinct, have the

same topology: one can be obtained from the other by a continuous transformation- realise the ellipsis as the shadow of a non-horizontal circle and consider the transformation along the rays of light. Every field in geometry comes with its underlying topological aspects, and the fundamental mathematical question is to understand the effect that the topology has on the geometry. In the case of Poisson geometry, although there have been several calls/attempts and there is a large potential for applications, the field of "Poisson topology" is still to be discovered.

"Symmetry": When looking at most of Escher's drawings, one realizes the presence of symmetry which allows us to reconstruct the complete drawing from a smaller part of it. More generally, one of the central ideas in geometry is to understand the objects via their "symmetries". Of capital importance for the modern geometry (and its applications), is the work of Lie (1890's) and Cartan (1900's) on symmetries of differential equations, which lead to many fundamental tools (such as Lie groups, pseudogroups, Lie groupoids, equivalence methods, etc) which pervade modern mathematics and theoretical physics.

1d. Former Vidi applications: no

1e. NWO Council area : EW

1f. Host institution: Utrecht University

Research proposal

2. Description of the proposed research

2a. Research topic *(an introduction into the mathematical background and the objectives)*

The present project has three components: a Poisson, a Cartan and a Lie component. In the *Poisson component*, the overall aim is the study of "Poisson topology"- a field still to be born. In the *Cartan component*, using our expertise in Poisson geometry and Lie algebroids, we aim at further understanding of Cartan's fundamental work and of the geometry of differential equations, with an eye on applications. The *Lie component* serves both as a tool for handling specific problems, and as an overall method for interactions of the various parts of the project with each other and with other fields. In this first part of the proposal we give an introduction into the mathematical landscape of the project :

we present the main keywords and the interactions between them, and, along the way, we will quietly indicate some research problems. We try to keep the presentation as non-specialized as possible, and we will also pay attention to the historical development- which we find relevant for the project.

Poisson geometry: Nowadays, Poisson geometry is known to be relevant to a whole range of fields in mathematics and mathematical physics, and each such field offers itself a different motivation for studying Poisson geometry. Its original motivation however comes from the fact that it provides the mathematical framework in which the basic constructions of Hamiltonian mechanics can be carried out. Classically, one works in the $2n$ -dimensional space of coordinates (x, p) , where “ x ” stands for the position and “ p ” stands for the momentum; the classical Poisson bracket is an operation on the space of hamiltonians (functions of x and p) given by:

$$\{f, g\} = \sum \left(\frac{\partial f}{\partial x_i} \frac{\partial g}{\partial p_i} - \frac{\partial f}{\partial p_i} \frac{\partial g}{\partial x_i} \right)$$

Poisson invented his brackets in 1809 as a tool, and later (1884), Jacobi realized the importance of these brackets and elucidated their algebraic properties- the Leibniz identity and the Jacoby equation. In general, a Poisson structure on a space (manifold) M is an operation $\{-, -\}$ on the space of functions on M , which satisfies these identities. We will come back later to our presentation of Poisson geometry.

Symplectic geometry: The standard Poisson structure we have just mentioned has the property that it is “nondegenerate”, which amounts to the fact that one can pass from the Hamiltonian formulation to the Lagrangian formulation and back. Symplectic geometry studies such nondegenerate Poisson structures. It has the remarkable fact that this geometry is trivial locally: any symplectic structure locally looks like the standard model mentioned above. However, the situation is dramatically different globally, and a large part of the theory comes under the name of “symplectic topology”. The modern symplectic geometry/topology began to take its modern shape with the formulation of the Arnold conjecture in the 1960's. Through its huge developments, symplectic geometry conquered an independent and rich territory, with striking applications to, and connections with, complex geometry, (low dimensional) topology, mathematical physics, to mention just a few [McDSa, Gro].

Foliation theory: A foliation of a given space is a partition of the space into subspaces (called leaves) which, locally, are nicely distributed (like the foliation of the usual 3-dimensional space by horizontal planes, or obvious generalizations of this). Given a foliation, one can talk about the longitudinal geometry (i.e. along the leaves) and the transversal geometry (i.e. perpendicular to the leaves). Some ideas of foliation theory go back to the nineteenth century to the work of P. Painleve on holomorphic differential equations. Also, the notion of leaf plays a fundamental role in the work of E. Cartan around 1900 (mentioned below). However, “foliation theory” began with the work of C. Ehresmann and G. Reeb in the 1940's [EhRe, Reeb] on the question of H. Hopf of whether one can find a foliation of the 3-dimensional sphere by leaves which are 2-dimensional; Reeb found such an example, while Haefliger showed that, if one requires analytic formulas, such foliations cannot exist [Hae1956]. Right from the start, foliation theory was strongly influenced by the question of how the topology influences the behaviour of the foliation (with notable results by Thurston and others). Nowadays, foliation theory is a well established field central to geometry.

Lie groups/algebras: Lie groups arise as transformation (symmetry) groups of various objects. The birth of Lie groups goes back to the work of Sophus Lie at the end of the nineteenth century and it has its origins in Klein's "Erlanger Programm" and in the study of differential equations [Lie1880, Lie1885]. Lie began his work in the 1870's in the context of transformation properties of differential equations, seeking for a "Galois theory" in this context, for integrating the equations [Lie1895a, Lie1895b]. One of Lie's greatest contributions, and one of his main tools, was the use of infinitesimal methods, which allows one to pass from complicated (nonlinear) conditions to much simpler (linear!) ones. The abstract formulation of Lie groups (and their infinitesimal counterpart, Lie algebras) reached its modern form in the first part of the nineteenth century with the work of H. Weyl [Weyl] around 1925 and of C. Chevalley [Chev] around 1946. Nowadays, Lie groups and Lie algebras pervade mathematics and theoretical physics.

Back to Poisson geometry: Back to our outline of the history of Poisson geometry, after the initial work of Poisson and Jacobi, it was Lie himself who began the study of the geometry of the Poisson brackets. In particular, some of Lie's formulas translate into the simple fact that Lie algebras are the same thing as a special class of Poisson structures-the "linear" ones. This is just an indication of the strong relationship between Lie theory and Poisson geometry. Despite this, and despite the fact that symplectic geometry was an active field of research already in the 1960-1970's, Poisson geometry became active only in the 1980's with the work of A. Weinstein [Wein1983], stimulated by connections with a number of areas, including harmonic analysis on Lie groups, infinite dimensional Lie algebras, singularity theory, integrable systems, to mention just a few examples.

From a mathematical point of view, Poisson geometry brings together the three different fields we have just mentioned: foliation theory, symplectic geometry, and Lie theory. More precisely, any Poisson structure on a space M induces a foliation on M , induces symplectic structures in the longitudinal direction (on the leaves) and Lie group/algebra structures in the transversal direction, all interacting with each other in a non-trivial manner.

It is interesting to point out that, in contrast with symplectic geometry and foliation theory, Poisson geometry is nontrivial even locally, and a large part of the research is motivated by such local issues. On the global side, a large part of the research in Poisson geometry was devoted either to the general theory, including the discovery of the deeper connections with Lie theory ([Kar, CoDaWe, CatFe, CrFe2004], to which we will come back later) or to more specific applications in other fields. Again, in contrast with symplectic geometry and classical foliation theory, and despite recent calls for such a theory, there is no such a field as "Poisson topology" (...).

Lie pseudogroups: Strictly speaking, what Lie was working with were not the modern (abstract) Lie groups but what we nowadays call Lie pseudogroups. The precise definition originates in Lie's outline of a general theory for integrating (partial) differential equations, based on the "structure" of the "group of local transformations" which leaves invariant the given equation [Lie1895a, Lie1895b]. In general, a Lie pseudogroup is a collection of local transformations of a given space, the transformations being characterized by the fact that they preserve a differential equation (or, equivalently, that they are the local solutions of such an equation). If a Lie group acts on our initial data, it gives rise to such

pseudogroups, but, among all pseudogroups, these are of a rather special type: they are those which depend on a finite number of parameters. However, it is for this kind of “finite continuous groups” that Lie was able to use his infinitesimal methods to write down the “structure equations”, etc. Although Lie believed that his methods were still appropriate in the general (infinite) situation, some of Lie’s methods turned out to be difficult to extend (at least for a while). So, although the early theory of Lie pseudogroups was established by Lie (and his student Tresse), essential progress (in the direction of structure equations) was made by Vessiot [Ves1903, Ves1904] and especially by E. Cartan [Car1905, Car1908, Car1937a, Car1937b].

Cartan's work: Cartan's success in handling Lie pseudogroups was based on the departure from one of the basic ideas of Lie: infinitesimal methods. Instead, he took an alternative (really “orthogonal”) point of view which lead him to what is known today [BCGG] as “the theory of exterior differential systems”. The foliations mentioned above provide the simplest examples of such systems. The main idea of Cartan is that any differential equation can be handled as an exterior differential system. This is a fundamental idea which goes beyond its original scope (the study of Lie pseudogroups) and has influenced the modern differential geometry substantially.

Still motivated by the study of Lie pseudogroups- namely the question of whether two such objects are isomorphic- Cartan invented his “method of equivalence” [Car1908, Car1937b] which, again, proved to be an important tool in geometry.

Understanding Cartan's ideas took a certain amount of time and considerable efforts, with important contributions from several prominent geometers who treated it from what became a modern geometrical point of view. Such examples are the work of Chern-1953 [Ch], Kumpera and Spencer-1970 [KuSp], Kuranishi-1961 [Kur], Singer and Sternberg- 1965 [SinSt], Guillemin and Sternberg [GuSt1964, GuSt1966], Molino, Pommaret 1978 [Pom]. Out of their contributions came for instance the theory of G-structures- central to differential geometry (which also influenced foliation theory). Also, what Singer and Sternberg did was, roughly speaking, a “conciliation” of Cartan’s methods with Lie’s infinitesimal point of view, but in the case of transitive Lie pseudogroups only [SinSt] (they never came back to the general case ...). It is interesting to point out that everybody agrees that reading from Cartan’s work is not an easy task, one has to use his imagination, and that makes it worth reading it again (...). Although Cartan's work is a cornerstone of geometry, it is a place where differential geometry, (homological) algebra and differential equations come together. In particular, there is nowadays an exciting and active field of “geometric study of differential equations” [BGH, Kam, Olv, Storm].

Lie groupoids/algebroids: Lie groupoids are another generalization of Lie groups, which is closely related to Lie pseudogroups, with the difference that Lie's infinitesimal methods can still be used directly; in particular, any Lie groupoid has an underlying infinitesimal object, called Lie algebroid. Another important advantage of Lie groupoids comes from the large number of examples arising naturally from different sources; this provides us with a basis for unification and interaction between various classical fields in mathematics, such as foliation theory, noncommutative geometry, gauge theory, Poisson geometry, equivariant geometry, orbifolds, the geometry of momentum maps, etc.

Historically, Lie groupoids were introduced in geometry by Ehresmann in the 1950's [Ehr], in relation with aspects related to the geometry of differential equations. One of his key remarks starts from the basic idea of Taylor approximation: given a function f , its polynomial-like approximation (which influences the behaviour of f) around x is determined by the values of f and of its partial derivatives at the given point x , with a well-known explicit formula-the Taylor expansion. In his global formulation of this principle, Ehresmann was led to the discovery of the language of jets which has been used ever since; intuitively, the jet of a function at a point is precisely the Taylor expansion of f at that point. When applied to functions f which are local transformations of a Lie pseudogroup on a space X , the resulting jets give rise to "collections of arrows (paths)" which "travel" between different points of X , with a simple mathematical property: two such paths can be composed if they match (i.e. if the end point of one coincides with the starting point of the other one), and this composition satisfies some simple conditions. This is precisely what a general Lie groupoid over X is, and what Ehresmann proposed was to forget about the starting Lie pseudogroup and only retain the resulting jets, i.e. Ehresmann's jet groupoids.

Lie groupoids continued to be used and studied in this context by Ehresmann's students and others (e.g. [Lib]), but the theory came to a halt (at least for a while) with the work of Kumpera and Spencer [KuSp]. The revival of the theory took place in stages, and very much depended on the discovery of connections with (and inputs from) other fields, such as foliation theory (Haefliger, Bott in the 1970's [BoHae, Hae1970]), noncommutative geometry (Connes in the 1980's [Connes]), gauge theory (Mackenzie in the 1980's [Mack]), algebraic topology and topos theory (Moerdijk [Moer]) and especially Poisson geometry (Weinstein's school, starting in the 1980's, [Kar, Wein1983, CaDaWe]). For instance, it is interesting to look at the path followed by one of the main questions in the theory, which is at the heart of Lie's principle- the integrability problem: when does a Lie algebroid come from a Lie groupoid? The question was first considered by Ehresmann's student J. Pradines [Prad]), who believed to have solved it (but without giving a complete proof). Later, foliation theory proved that Pradines was mistaken, producing the first nonintegrable examples [AlmMol], while Mackenzie, coming from gauge theory, discovered first obstructions. Ultimately, it was Poisson geometry which had the strongest influence [CaDaWe, CatFe], eventually leading us to a complete solution [CraFe2003].

It is also interesting to note the not so well-known fact that the first occurrence of Lie algebroids and of the integrability problem can be found already in the work of Cartan [Bry] This is an intriguing point to which we will return in the second part of the project

Back to Poisson geometry: The relation between Poisson geometry and Lie theory, although clear at an early stage, turned out to be much deeper than initially thought [Kar, CaDaWe, CatFe, CraFe2004]. First of all, Lie algebroids can be seen as particular cases of Poisson structures (those which are "fiberwise linear"). Conversely, and more interestingly, a Poisson structure gives rise to a Lie algebroid which, after an integration step (mentioned above), produces a Lie groupoid which is a ... symplectic manifold! Hence, using Lie groupoids we obtain a "symplectization functor"

$$\Sigma : \text{Poisson Geometry} \longrightarrow \text{Symplectic Geometry}$$

Poisson geometry and Lie groupoids strongly influenced each other, up to the present point when we

realized that they always come together. According to the general philosophy, this point of view also brings new tools and ideas into Poisson geometry, coming from other fields where Lie groupoids are relevant (e.g. foliation theory, noncommutative geometry, equivariant geometry).

Example of the interaction: local issues. As we have already mentioned, Poisson geometry is nontrivial even locally. Even more, the local study of Poisson structures sometimes require an understanding of phenomena which are rather global, and that is because the leaves may still be topologically nontrivial in arbitrarily small neighbourhoods (think for instance of the foliation of the space by concentric spheres approaching the centre). Hence, there is often interaction between the local and the global Poisson geometry. Positive local results in Poisson geometry are quite remarkable and highly nontrivial. A very good example is the following result of J. Conn: if the transversal Lie group at a singular point (i.e. a leaf of zero dimension) is "rigid" (i.e. compact) then, around that point, the Poisson structure looks like the linear Poisson structure associated to the transversal Lie algebra [Conn]. Conn's proof is analytical and full of estimates, and there have been repeated attempts aiming at finding a more geometric proof [Wein2000]. These attempts generated themselves important results, some of which went beyond the original aim and beyond Poisson geometry. Back to Conn's result, we believe that we are now within reach of such a geometrical proof, outlined already in [CraFe2005], and whose completion inevitably becomes a part of this project. It is very instructive to point out the main steps of such a proof, since it clearly shows the interactions between the various fields. First, one uses a technique borrowed from symplectic geometry (Moser trick) and the language of algebraic topology to reduce the problem to a cohomological one. Then, an integration step (in the sense mentioned above) provides us with a Lie groupoid. Next, we borrow a technique from Lie groups (averaging and the van Est isomorphism [Crai]) to solve the cohomological problem. In the integration step we first had to prove a stability result [CraFe2007] which is inspired by similar results in foliation theory [Thur] and equivariant geometry [Stow]. At this point we expect that, even more, such a stability result is true for general Lie algebroids. This is another (type of) question(s) for this project. Such a result would not only provide a unification, but also new understanding/proofs of the classical results, including the well known Thurston's stability in foliation theory.

Back to the local geometry of Poisson manifolds, it is interesting to point out that a Poisson structure can be viewed locally as a solution of the (system of) differential equations

$$\sum_{cyclic(i,j,k)} \sum_u \pi_{iu}(x) \frac{\partial \pi_{jk}(x)}{\partial x_u} = 0$$

This is the kind of differential equation which does not fit into the types of equations one usually studies (in relation with Cartan's work). Putting it differently, the resulting pseudogroup is, strictly speaking, not a Lie pseudogroup. However, such kind of equations do appear and are interesting. This indicates another class of problems for the present proposal. It is instructive to look again at Conn's result mentioned above, from this point of view. It says that, under certain conditions (π vanishes at the origin, and the partial derivatives of π at the origin take certain prescribed values coming from Lie theory), there is essentially a unique solution. Moreover, one may say that Conn's proof handles the differential equation itself. Another interesting fact here: the 3-dimensional case of this result can be

interpreted as a result about exterior differential systems (just a “Pfaff form”), and this particular case was discovered already by ... Reeb in his thesis [Reeb] (where foliation theory was being born). And there is yet another way to look at Conn’s result, namely as an instance of Cartan’s equivalence problem. Again, although the usual technical (regularity) requirements are not satisfied, Conn’s success shows that there are new classes of equations which can be handled, raising new questions for this project, at the border between geometry and differential equations.

2b. Approach *(the main components of the project, with more specific directions)*

We now discuss the main components of the project and specific directions.

Poisson component: On the Poisson side, we plan to focus on “Poisson topology”. As we have already mentioned, very little is known in this direction. However, our results on stability mentioned above [CrFe2007], as well as recent work on “Poisson fibrations” [Fer] already reveal deep topological implications.

One approach departs from symplectic topology: how do the known “symplectic phenomena” [McDSa] extend to the Poisson setting? One of the simplest particular cases of this is: how does symplectic topology vary under deformations? This is already non-trivial, and it alone can have interesting consequences. The “symplectization functor” (see 2a) should be an useful tool here.

Another approach departs from Lie group theory. For instance, we now understand what a “Poisson compact” space should be (playing a role similar to that played by compact groups in Lie theory). Finding interesting examples is itself a non-trivial matter- for instance, finding examples in the lowest possible dimensions brings us to a known problem in symplectic topology, which has been open for a while [Kot]. In general, using the recent exciting results of T. Zung, we know that the space of leaves of such a “Poisson compact” space is locally polyhedral with an induced integral affine structure [Zung], pointing into the direction of classification results similar to the ones of Delzant in symplectic geometry [Del](proving such a result would already be a major achievement). This, together with the example mentioned above, points to yet another connection, namely with the momentum map theories of [AMM, CraBu].

Finally, another approach for studying Poisson geometry in the context of the present project is via Cartan-like methods, as we have already hinted in 2a. Cartan and his followers worked in a more restricted context, but one may try to use some of their ideas. This part may also raise questions of a rather analytic nature about certain types of differential equations, where Nash-Moser techniques [Ham] may be needed.

Lie component: This part of the project refers to the general theory of Lie groupoids/algebroids and of Lie pseudogroups. We plan to use them both as an overall method (they serve as a “carrier” for transporting ideas/techniques from one field to another) and as a world where some of the questions from the other parts of the project live and can be handled more conceptually. The first philosophy has been already encountered in the stability theorem mentioned in 2.a (the Poisson component of the project): the idea of such a result came –via Lie algebroids- from foliation theory [Thur] and equivariant geometry [Stow]. We now believe that, we can handle the question for all Lie algebroids, producing an

unifying proof which is new even for the classical results such as Thurston's stability. The method we have in mind uses our cohomological study [CraMoe] and an implicit function theorem in a Banach setting (with control of Sobolev type).

Also the Cartan component of the project will bring in several research problems. For instance, one has to understand a certain notion of "Cartan equivalence" of groupoids which comes from Cartan's notion of "equivalence holoedrique". Also, multiplicative exterior differential systems on groupoids seem to be relevant (hence have to be defined and studied), and our study of the particular case of 2-forms [CBWZ] should be helpful. There are certain cohomological aspects which have to be understood (coming from Spencer type cohomologies [KuSp, BrGr]), and we hope to use here our previous work [Crai, CraMoe]. Also, certain types of "bisections" of groupoids will probably have to be studied in connection with "Cartan's standard form" (structure equations) of Lie pseudogroups. And the list can continue for a while.

Cartan component: An overall objective in this part of the project is to find the precise relations between Lie algebroids and Cartan's work (this will surely lead to further understanding of Cartan's work and geometrical applications), and to go back to the original motivation of Cartan and Lie- the geometry of differential equations. The fact is that there are several striking (implicit) occurrences of Lie algebroid/groupoid structures, and even of the integrability problem, in the work of Cartan and his followers. For instance, a particular type of "abstract structure functions" coincide with a Lie algebroid structure, and realizing these functions as concrete structure functions coincides with the (local) integrability problem. This is pointed out in [Bry], where it is beautifully used. In the same direction, in the equivalence problem, Lie algebroids arise when reducing the number of parameters in the structure functions (as e.g. in [Olv]) or when putting the Lie pseudogroups in "standard forms" (as e.g. in [Kam1989]). On the other hand, the work of Guillemin, Singer and Sternberg that we mentioned in 2a still has to be continued (the non-transitive case), and we expect that Lie groupoids/algebroids (or techniques that come from that direction) will be useful. This sequence of implicit occurrences continues further in the direction of the geometry of differential equations, their infinitesimal symmetries [AKO, Olv, Kam2002, etc]. The list of such quasi-speculative comments can go on for a while, calling for more fundamental research in this direction; a good understanding of these aspects would undoubtedly be rewarding.

2c. Innovation

- a systematic study of the "variation of symplectic topology" under deformations
- the use of the symplectization functor to study Poisson topology, as well as the study of "Poisson compact" spaces which came up very recently in joint discussions with R.L. Fernandes and D.M. Torres.
- the idea of using "Cartan's methods" in Poisson geometry is new.
- the use of Lie algebroids to attack in a unified way stability properties of leaves is new.
- a systematic review of some of Cartan's work from the point of view of Lie algebroids, as we intend it (going beyond Ehresmann-type interpretations and [Pom]), we believe is new.

2d. Plan of work

Research team:

- the principal investigator
- one PhD student for 4 years, to work on the Poisson topology side of the project. We hope to recruit one of the students of the ongoing Master Class coordinated by the principal investigator. Such a student would already have a considerable background (the theme of the Master Class is “Symplectic geometry and beyond”).
- one PostDoc for 3 years or one PhD student for 4 years. The favourite choice is a PostDoc with solid background on Cartan’s methods/the geometry of differential equations, to help on the Cartan side of the project. If we cannot find such a PostDoc, we will appoint a second PhD student, to work in the same direction.

Collaborators/contacts:

- local contacts are E. van den Ban (Lie theory), H. Duistermaat (symplectic topology, differential equations), E. Looijenga (pseudo-holomorphic curves), J. van de Leur (infinite dimensional Lie algebras), I. Moerdijk (Lie groupoids, algebraic topology), David Martinez Torres (symplectic topology).
- international contacts: R.L. Fernandes (Lisbon), T. Zung (Toulouse), A. Alekseev (Geneva), E. Meinrenken (Toronto), H. Bursztyn (IMPA, Brasil), A. Weinstein (Berkeley). We hope to establish new contacts with the school of R. Bryant and P. Olver.

Timeline/plan of work:

- in the first part the principal investigator will continue the work with R.L. Fernandes on the general stability result (part of the Lie component) and the geometric proof of Conn’s theorem.
- at the same time, the study of “Poisson compact” spaces will be started by the principal investigator in collaboration with R.L. Fernandes and D.M. Torres. Some aspects will probably be discussed with T. Zung. If the Lie-group valued momentum map turn out to be relevant, we may contact A. Alekseev, E. Meinrenken and H. Bursztyn. This may cause further research of Dirac valued momentum map (such as our work [CraBu]). Some of these collaborations will require some short visits (organizing a workshop may be appropriate). The precise development of this part of the research is very difficult to predict, and may very well continue over the entire period of five years. It may also develop a PhD subproject.
- the first PhD student will belong to the Poisson component of the project. In the first year, he will be guided through some of the existing literature on symplectic geometry (we have in mind the monograph [McDSa] and Gromov’s paper [Gro]). In this first year, he will also meet regularly with D.M. Torres (for acquiring background in differential topology), and we may organize a reading seminar. In the second year, the student will start the Poisson topology study. Depending on the progress in the study of “Poisson compact” spaces, he will be asked to work on deformations aspects of symplectic topology, with an eye on the effect of the symplectization functor. More concrete plans will depend on the achievements and the findings along the way.
- the work on the Cartan component will be done by the principal investigator. In this direction, I hope that certain aspects will be discussed with H. Duistermaat. If we manage to appoint a PostDoc, he will

join the research, and he will start with giving a minicourse on the relevant topics. If a PhD student is appointed, the first year will be devoted to acquiring the relevant background, and we first organize a reading seminar on [BCGG], with an eye on the original papers of Cartan [Car1905, 1908, 1910, etc]. The second year he will be asked to look at Cartan's structure equations and clarify the role that algebroids (and possible generalizations) play there. The next step depends on his findings, while in the last part we hope he will be looking at the relevance of his findings to the geometric study of differential equations.

- along the way, problems in the Lie component of the project which arise from the other parts will probably be discussed with I. Moerdijk. Cohomological aspects will probably be discussed also with C.A. Abad.

- in the first stage, the study of intransitive Lie pseudogroups will be done separately from the rest of the project (but interactions will surely follow). The principal investigator will also keep watching for the influence that some parts of the project may have on each other.

2e. Literature references

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2f. Utilisation paragraph: Only required for proposals to be submitted to Technical Sciences (STW), see Notes.

Cost estimates

3a. Budget

	200y	200y+1	200y+2	200y+3	200y+4	TOTAL
Staff costs: (in k€)						
Applicant	45	45	45	45	45	225
Post-doc		56	57.5	59.5		173
PhD student	40	42	44	46		172
Support staff						
Non-staff costs: (k€)						
Equipment						
Consumables						
Travel and subsistence	6	6	6	6	6	30
Other						
TOTAL						600

Notes:

- The budget plan has been prepared in agreement with the institute.
- The Institute will also contribute with the difference to the full salary of the applicant (an extra amount of about 194 k).
- The "travel and subsistence" estimation contains the travel costs, the costs for buying books and the costs for inviting visitors.

3b. Indicate the time (percentage of fte) you will spend on the research: 1fte

3c. Intended starting date: September 1, 2007

3d. Have you requested any additional grants for this project either from NWO or from any other institution? no

Curriculum vitae

4a. Personal details

Title(s), initial(s), first name, surname: Marius Crainic

Male/female: male

Date and place of birth: 03/02/1973

Nationality: romanian

Birth country of parents: Romania

4b. Master's ('Doctoraal')

University/College of Higher Education: Mathematical Research Institute, The Netherlands

Date (dd/mm/yy): 30/06/1996

Main subject: algebraic topology, K-theory, number theory

4c. Doctorate

University/College of Higher Education: Utrecht University

Date (dd/mm/yy): 03/04/2000

Supervisor ('Promotor'): I. Moerdijk

Title of thesis: Cyclic cohomology and characteristic classes for foliations

4d. Work experience since graduating

- April 2000- September 2002: Post Doc at the Mathematical Institute, Utrecht University (with an unpaid leave during October 2001- September 2002). This position was a temporary, full time position.

- October 2001- September 2002: Miller Research Fellow, University of California at Berkeley. This has been a temporary, full time position. I was involved in the supervision of the PhD Thesis of Chenchang Zhu (student of Alan Weinstein).

- September 2002- present: Research Fellow of the Dutch Royal Academy (KNAW) at the Mathematical Institute, Utrecht University (the KNAW fellowship will end in September 2007). The present position in the department is "UD" (Universiteit Docent), which is a permanent, full time position. I am involved in the supervision of a PhD student (Camilo Arias Abad) and of a PostDoc (David Martinez Torres).

4e. Man-years of research: 6 years and 9 months

4f. Brief summary of research over last five years

The main directions of the research in the last five years are the following (note: some of them have

been joint projects, cf. the list of publication):

- proved the integrability theorem for Lie algebroids [13] and applications to Poisson geometry [9], generalized complex geometry [22], prequantization [8], Dirac geometry [10] and Jacobi structures [7].
- Lie-type approach to Poisson geometry, including the study of the symplectization functor and solving the existence of complete symplectic realizations problem [9, 12].
- study of momentum map theories, introducing the “Dirac geometry framework”, with applications to Lie group valued momentum maps [5, 21].
- advances in finding a geometric proof of Conn’s linearization theorem [2, 9, 12].
- systematic study of cohomologies of Lie algebroids [6, 12], with ongoing research on Weil and Cartan models (joint with my PhD student, C.A. Abad).
- study of deformations of various geometric structures and rigidity phenomena [2, 6, 20, 23].
- study of characteristic classes [4, 11, 12].
- aspects of of foliation theory and noncommutative geometry, which also inspired and/or motivated some of the research already mentioned.

4g. International activities

- October 2001- June 2002: Miller Research Fellow, University of California at Berkeley. Working with A. Weinstein, and starting cooperation with H. Bursztyn (presently at IMPA, Brasil), Chenchang Zhu (presently at Institut Fourier Grenoble).
- March 2002: Visit at Universite Paul Sabatier (Toulouse, France), where I gave a minicourse.
- September 2002: Visit at the Erwin Schrodinger Institute (Vienna, Austria), related to the program “Aspects of Foliation Theory in Geometry, Topology and Physics”.
- May 2003: “Invited Professor” , Clairmont-Ferrant, France.
- August 2003: Visit at the Erwin Schrodinger Institute (Vienna, Austria), related to the program “Moment maps and Poisson geometry”.
- July 2005: Visit at the ICTP (Trieste, Italy), where I gave a minicourse together with R.L. Fernandes.
- October 2005: Visit to the University of Geneva (invited by A. Alekseev and A. Heafliiger).

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- 2001- 2006: several participations in workshops organized at Mathematisches Forschungsinstitut Oberwolfach, Germany (participation is only by invitation).
 - 2001- 2006: invited to various other conferences, such as: Poisson 2002 (Lisbon, September 2002), PQR 2003 (Brussels, June 2003), Poisson 2004 (Luxembourg, June 2004), Conference on Poisson Geometry ICTP (Trieste, July 2005), Poisson geometry in mathematics and physics (Tokyo, June 2006).
 - 2001- 2006: several 2-weeks visits to Instituto Superior Tecnico, Lisbon, Portugal, working with R.L. Fernandes.
 - 2006: co-organizer of the Oberwolfach meeting "Poisson Geometry and applications" (April 29-May 5, 2007).
 - 2007-2008: "Invited Professor" for one month, giving to a minicourse in the "Groupoids and Stacks in Physics and Geometry" program, IHP, Paris (Jan 8 - April 6, 2007). Invited at Centre international de recherches en mathematiques (France) in September 2007, to give a minicourse on groupoids and dynamical systems. Invited at the Centre de Recerca Matematica (Barcelona) for one month (May 2008) in the program "Geometric Flows and Equivariant problems in Symplectic Geometry".

4h. Other academic activities

- Coordinator of the Master Class (since 2004), which is a project of the Dutch Mathematical Research Institute to bring to Holland young talented students. In particular, I am involved in attracting new funds to support the foreign students- with the main achievement that the Master Class is now associated with the various NWO clusters (for instance, for a period of 5 years, the GQT cluster provides an amount of 80 000 euros/year to support the program). Also, I am actively working on new ways of attracting more students with a promising potential, aiming at creating "Master Class points" for recruiting such students.
- Organizer of the Stafcolloquium at Utrecht University (since 2005).
- Member of the editorial board of the journal "Mathematica", Romania (starting in 2007).
- organized/co-organized various local and national seminars.
- Scientific organizer of the 2006/2007 Master Class "Symplectic geometry and Beyond".
- involved in the scientific organization of the 2003/2004 Master Class on Noncommutative geometry.
- organized the workshop "Alan's day" , in relation with the honorary doctorate awarded by the

University of Utrecht to Alan Weinstein on March 26, 2003.

4i. Scholarships and prizes

- 2004: NWO (Dutch National Science Foundation) Open Competitie project “Symmetries and Deformations in Geometry” was awarded. This funds a PhD position (4 years) and a PostDoc position (3 years). Amount of money involved: 352.5 k euro.

- 2002: KNAW (Dutch Royal Academy) Research Fellowship for the period 2002-2007, The Netherlands. Amount of money involved: 305 k euro.

- 2001: Miller Research Fellowship for the period 2001-2004 (interrupted half way to start the KNAW position), University of California at Berkeley. Amount of money involved: around 150 k euro.

List of publications

5. Publications:

I. Proceedings of international conferences (refereed):

1. *Lectures on integrability of Lie brackets*, joint work with R.L. Fernandes, Proceedings of "Poisson 2005 ICTP" (2007).
2. *Rigidity and flexibility in Poisson geometry*, joint work with R.L. Fernandes, Travaux mathematiques. Fasc. XVI, 53--68, Trav. Math., XVI, Univ. Luxemb., Luxembourg (2005).
3. *Quasi-Poisson structures as Dirac structures*, joint work with H. Bursztyn and P. Severa, Travaux mathematiques. Fasc. XVI, 41--52, Trav. Math., XVI, Univ. Luxemb., Luxembourg (2005).
4. *Secondary characteristic classes of Lie algebroids*, joint work with R.L. Fernandes, Quantum field theory and noncommutative geometry, , Lecture Notes in Phys. 232(2005), pp. 157-176.
5. *Dirac structures, momentum maps, and quasi-Poisson manifolds*, joint work with H. Bursztyn, The breadth of symplectic and Poisson geometry, Progr. Math. 157 (2005), pp. 1-40.

II. International refereed journals

6. *Deformations of Lie brackets: cohomological aspects*, joint work with I. Moerdijk, to appear.
7. *Integration of Jacobi manifolds*, joint work with Chenchang Zhu, to appear in Journal de l'Institut Fourier.
8. *Prequantization and Lie brackets*, J. Symplectic Geom. 2 (2004), pp. 579-602.
9. *Integrability of Poisson brackets*, joint work with R.L. Fernandes, J. Differential Geom. 66 (2004), pp. 71-137.
10. *Integration of twisted Dirac brackets*, joint work with H. Bursztyn, A. Weinstein, Chenchang Zhu, Duke Math. J. 123 (2004), pp. 549-607.
11. *Cech-De Rham theory for leaf spaces of foliations*, joint work with I. Moerdijk, Math. Ann. 328 (2004), pp. 59-85.
12. *Differentiable and algebroid cohomology, van Est isomorphisms, and characteristic classes*, Comment. Math. Helv. 78 (2003), pp. 681-721.

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13. *Integrability of Lie brackets*, joint work with R.L. Fernandes, *Annals of Mathematics* 157 (2003), pp. 575-620.
14. *Cyclic cohomology of Hopf algebras*, *J. Pure Appl. Algebra* 166 (2002), pp. 29-66.
15. *Foliation groupoids and their cyclic homology*, joint work with I. Moerdijk, *Advances in Mathematics* 157 (2001), pp. 177-197.
16. *A homology theory for etale groupoids*, joint work with I. Moerdijk, *J. Reine Angew. Math.* 521 (2000), pp. 25-46.
17. *Cyclic cohomology of etale groupoids: the general case*, *K Theory* 17 (1999), pp. 319-362.
18. *On two-primary algebraic K-theory of quadratic number rings with focus on K_2* , joint work with P. A. Ostvaer, *Acta Arithmetica* 87 (1999), pp. 223-243.
19. *A note on the denseness of complete invariant metrics*, joint work with V. Csaba, *Publ. Math. Debrecen* 51 (1997), pp. 265-271.

III. Others

20. *Stability in Poisson geometry*, joint work with R.L. Fernandes, submitted.
21. *Dirac geometry and quasi-Poisson actions*, joint work with H. Bursztyn, submitted.
22. *Generalized complex structures and Lie brackets*, submitted.
23. *On the perturbation lemma, and deformations*, submitted.
24. *Birkhoff Interpolation with Rectangular Sets of Nodes*, submitted.

Signature

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Place: Utrecht

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Place: Utrecht

Date: 2007-01-10

Postal address:

Mathematical Instituut,
Utrecht University, PO Box 80.010
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