Solutions Book Chapter 13 (Continued), SCI 113 Spring 2008

(1) **Exercise 13.12** Eigenvalues are $\lambda_1 = -5$, $\lambda_2 = -1$ and $\lambda_3 = 0$. Corresponding eigenvectors are $t\begin{pmatrix} 7 \\ -5 \\ 1 \end{pmatrix}$, $t\begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}$, and $t\begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix}$. The matrix $C = \begin{pmatrix} 7 & -1 & -1 \\ -5 & -1 & 0 \\ 1 & 1 & 2 \end{pmatrix}$ satisfies

$$C^{-1}AC = \begin{pmatrix} -5 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

(2) **Exercise 13.13** The matrix $C = \begin{pmatrix} -2 & 2 \\ 1 & 1 \end{pmatrix}$ diagonalizes A since

$$C^{-1}AC = \begin{pmatrix} -3 & 0 \\ 0 & 5 \end{pmatrix}.$$

(3) **Exercise 13.14** $C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & -1 \\ 0 & 1 & 2 \end{pmatrix}$, and

$$C^{-1}AC = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

(4) **Exercise 13.15** First note that if $D = \begin{pmatrix} \lambda_2 & 0 & 0 & \cdots & 0 \\ 0 & \lambda_2 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & \lambda_n \end{pmatrix}$ is an $n \times n$ diagonal matrix, then $\det(D) = \lambda_1\lambda_2\cdots\lambda_n$. If $C^{-1}AC = D$, then $AC = CD$, hence $\det(AC) = \det(A)\ \det(C) = \det(C)\ \det(D) = \det(CD)$. Since C is an invertible matrix $\det(C) \neq 0$. Dividing both sides by $\det(C)$ we get that $\det(A) = \det(D) = \lambda_1\lambda_2\cdots\lambda_n$.

(5) **Exercise 13.28** Eigenvalues of A are $\lambda_1 = 4$, $\lambda_2 = 1$ and $\lambda_3 = -1$. The matrix C is given by $C = \begin{pmatrix} 2 & -1 & -3 \\ 2 & -1 & 3 \\ 2 & 2 & 0 \end{pmatrix}$, and

$$C^{-1}AC = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

(6) **Exercise 9.4.3 Lecture Notes** (1) Matrix M corresponds to S_1, matrix N corresponds to S_2, and matrix L corresponds to S_3. (2) Stable distribution= components of the eigenvector corresponding to the dominant eigenvalue. For matrix L the stable distribution is $\begin{pmatrix} 16 \\ 2 \\ 1 \end{pmatrix}$, for matrix M

the stable distribution is $\begin{pmatrix} 8 \\ 2 \\ 1 \end{pmatrix}$, and for matrix N, the stable distribution is $\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$. (3) Under matrix L we have

$$\begin{pmatrix} N_1(n) \\ N_2(n) \\ N_3(n) \end{pmatrix} = \frac{1}{2^n} \begin{pmatrix} 16 \\ 2 \\ 1 \end{pmatrix}.$$

Under matrix M we have

$$\begin{pmatrix} N_1(n) \\ N_2(n) \\ N_3(n) \end{pmatrix} = \begin{pmatrix} 8 \\ 2 \\ 1 \end{pmatrix}.$$

Under matrix N we have

$$\begin{pmatrix} N_1(n) \\ N_2(n) \\ N_3(n) \end{pmatrix} = \frac{1}{2^n} \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}.$$