Assignment 1, SCI 113 Spring 2008
Deadline Thursday February 28

- The assignment should be handed in on paper, and not by email. Do not forget to put your name on it.
- Each student should hand in his/her own assignment. It is not allowed to hand in assignments with joint authorship.
- Do not just give the final solution to a problem. Provide full argumentation in clear sentences. The argumentation should clarify the steps followed in your reasoning and calculations. In particular, it is not enough to just say that Mathematica provided the answer. Of course Mathematica may be used as a tool to get ideas for a solution. However, Mathematica will never provide the final argumentation. It is your job to find it and to present it in detail.

(1) (a) Solve the following system of equations using *Gauss elimination method*:

$$\begin{cases} x + 2y + 3z = 5 \\ 2x + 3y + z = 1 \\ 3x + y + 2z = 6. \end{cases}$$

(b) Show that the following system has infinitely many solutions

$$\begin{cases} 2x + y = 2 \\ y - 2z = 3 \\ 2x + 2y - 2z = 5. \end{cases}$$

Show that there exists a vector $\mathbf{a} = (a_1, a_2, a_3)$ and $\mathbf{b} = (b_1, b_2, b_3)$ such that all solutions $\mathbf{v} = (x, y, z)$ of the above system are given by $\mathbf{v} = t\mathbf{a} + \mathbf{b}$, with $t \in \mathbb{R}$. What does this mean geometrically?

(2) (a) Find the equation of the plane passing through the point $(1, 1, 1)$ and is perpendicular to the vector $\mathbf{u} = (2, -1, 3)$.

(b) Determine the equation of the plane P passing through the points $(1, -1, 2)$, $(2, 0, 1)$ and $(0, 1, 1)$. Find a vector $\mathbf{u}$ which is perpendicular to the plane P.

(3) (a) Express the complex number $(1 - \sqrt{3}i)^{-10}$ in standard form $a + ib$ with $a, b \in R$.

(b) Determine all solutions of the equation $z^5 = 32i$. Describe the solutions in polar coordinates. Sketch the solutions in the complex plane.

(c) Use De Moivre's Theorem to find an expression for $\sin 4\theta$ in terms of $\sin\theta$ and $\cos\theta$.

(4) Do problem 6.17 on p.142 of the textbook Mathematical Techniques by D.W. Jordan and P. Smith.

(5) Read section 7.4 of the book Mathematical Techniques, then do exercise 7.17 on p.161.