Geometric Mechanics

The last two exercises are homework, to be discussed on 9 March.

1). Let $A \in M_{n\times n}(\mathbb{R})$ be a matrix for which all eigenvalues are different from each other. Show that the vector space $\mathbb{R}^n$ admits the splitting

$$\operatorname{im} A \;\oplus\; \ker A \;=\; \mathbb{R}^n$$

as a direct sum of two A–invariant subspaces.

2). Compute the normal forms of order 3 and 4 of

$$H(x,y) \;=\; \frac{y^2}{2} \;-\; \cos x$$

with respect to the linear part H_0^0 of H.

3). Find the general solution $u = u(t,x)$ of the wave equation

$$\frac{\partial^2 u}{\partial t^2} \;=\; \omega^2 \frac{\partial^2 u}{\partial x^2} \tag{1}$$

on $\mathbb{R}^2$. *Hint:* consider compositions of scalar functions $f : \mathbb{R} \longrightarrow \mathbb{R}$ with linear combinations in x and t.

4). A vibrating string is modelled as the solution of the wave equation (1) on $\mathbb{R} \times [0,\pi]$ with boundary conditions $u(t,0) \equiv 0 \equiv u(t,\pi)$. Determine the general solution.
Hint: make a separation ansatz $u(t,x) = f(t) \cdot \psi(x)$.