Homework 10, Models of Intuitionism — Bobby Vos & Ruben Meuwese

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Exercise 1

Prove lemma 1 of Lifschitz (1979): there exists a unary partial recursive function α such that for every e, if $|V_e| = 1$ then $\alpha(e)$ is defined and $\alpha(e) \in V_e$. *(2 points)*

Exercise 2

In this exercise we examine two instances in which **HA** + CT_0 contradicts classical logic.

a) Show that, for an appropriate choice of formula $A(x)$, the sentence $\forall x(\neg A(x) \vee \neg\neg A(x))$ is not derivable in **HA** + CT_0. *(2.5 points)*

For the second contradiction, the following may prove useful:

b) Show that the sets $A = \{x \mid \exists y(Txxy \wedge U(y) = 0)\}$ and $B = \{x \mid \exists y(Txxy \wedge U(y) = 1)\}$ are recursively inseparable. *(2.5 points)*

Now, we have the following:

c) Show that, for an appropriate choice of functions α, β, the sentence

$$\forall x(\neg(\exists y(\alpha(x,y) = 0) \wedge \exists y(\beta(x,y) = 0)) \rightarrow \neg\exists y(\alpha(x,y) = 0) \vee \neg\exists y(\beta(x,y) = 0))$$

is not derivable in **HA** + CT_0. *(3 points)*