Homework 6, Models of Intuitionism — Bobby Vos & Menno de Boer

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Exercise 1

For each of the following sentences in **HA**, determine whether or not it is realizable. If a sentence is realizable, explain how to obtain a realizer.

a) $\forall x(S(x) \neq 0)$ *(1 point)*

b) $P \vee \neg P$, with P atomic *(1 point)*

c) $\forall x(x = 0 \vee \exists y(x = S(y)))$ *(1.5 points)*

Exercise 2

Show that there exists an instantiation of CT_0 that is not derivable in **HA**. *(2 points)*

Exercise 3

In this exercise you will prove Proposition 1.12 from the hand-out. Let F be an instance of ECT_0 i.e. $F = \forall x(A(x) \to \exists y B(x,y)) \to \exists e \forall x(A(x) \to B(x, \varphi_e(x)) \wedge \varphi_e(x) \downarrow)$ for some almost negative $A(x)$.

a) *(1.5 point)* Suppose e **rn** $\forall x(A(x) \to \exists y B(x,y))$. Show that

$$\forall x, n(n \textbf{ rn } A(x) \to \mathrm{snd}(\varphi_{\varphi_e(x)}(\psi_A(x))) \textbf{ rn } B(x, \mathrm{fst}(\varphi_{\varphi_e(x)}(\psi_A(x)))) \wedge \varphi_{\varphi_e(x)}(\psi_A(x)) \downarrow)$$

b)*(2 point)* Define $t_1(e) := \lambda x.\, \mathrm{fst}(\varphi_{\varphi_e(x)}(\psi_A(x)))$. Construct a term $t_2(e)$ which realizes

$$\forall x(A(x) \to B(x, \mathrm{fst}(\varphi_{\varphi_e(x)}(\psi_A(x)))) \wedge \mathrm{fst}(\varphi_{\varphi_e(x)}(\psi_A(x))) \downarrow)$$

(Hint: Use that $C(\varphi_x(y)) \wedge \varphi_x(y) \downarrow$ stands for $\exists z(T(x,y,z) \wedge C(U(z)))$).

c) *(1 point)* Conclude that $\mathbf{HA} \vdash \exists x(x \textbf{ rn } F)$.