Seminar on Set Theory

Hand-out lecture 13

December 18, 2015

Defintion 1 *A complete Boolean algebra B is called* countably Completely Generated *(ccg) if there is some countable $X \subseteq B$ such that B is the least complete subalgebra of B containing X.*

Theorem 2 *Suppose λ is an infinite cardinal. Then $B = RO(\lambda^\omega)$ is ccg and $|B| \geq \lambda$.*

Defintion 3 *Suppose A and B are Boolean algebras. A can be* completely embedded *in B if there exists a complete injective homomorphism from A to B.*

Theorem 4 *Let A be a Boolean algebra of infinite cardinality κ. Then A can be completely embedded in the collapsing $(\aleph_0, 2^\kappa)$-algebra.*

Theorem 5 *(**Rasiowa-Sikorski**) If S is a countable family of subsets of a Boolean algebra B and every member of S has a join, then for each $a \neq 0_B$ in B there is an S-complete ultrafilter in B containing a.*