Hand-in Exercise 2 - O-minimal Structures

3 oktober 2014

Problem 1.
Let F denote an ordered field and let R be an nontrivial ordered F-linear space as defined in (7.2). Construe R as a model-theoretic structure for the language $L_F = \{<, 0, -, +\} \cup \{\lambda\cdot : \lambda \in F\}$ of ordered abelian groups augmented by a unary function symbol $\lambda\cdot$ for each $\lambda \in F$, to be interpreted as multiplication by the scalar λ. Prove:

1. The subsets of R^m definable in the L_F-structure R using constants are exactly the semilinear sets in R^m.

2. The maps $R \to R$ that are additive and definable using constants are exactly the scalar multiplications by elements of F. A map f is additive iff

$$\forall r_1, r_2 \in R : f(r_1 + r_2) = f(r_1) + f(r_2).$$