Trends in Bifurcation Software:
From CONTENT to MATCONT

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Contents

- Generations of bifurcation software
- Features of AUTO, CONTENT and MATCONT
- Demos
- Perspectives

References

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$$\frac{du}{dt} = f(u, \alpha), \quad u \in \mathbb{R}^n, \alpha \in \mathbb{R}^m$$

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III: Closed environments (DsTool, CONTENT)

IV: Open environments (MATCONT, ...)

Features of AUTO, CONTENT, and MATCONT

	A	C	M
time-integration		+	+
Poincaré maps			+
continuation of equilibria	+	+	+
detection of branch points and codim 1 bifurcations (limit and Hopf points) of equilibria	+	+	+
computation of normal forms for codim 1 bifurcations of equilibria		+	+
continuation of codim 1 bifurcations of equilibria	+	+	+

Features of AUTO, CONTENT, and MATCONT

	A	C	M
detection of codim 2 equilibrium bifurcations (cusp, Bogdanov-Takens, fold-Hopf, generalized and double Hopf)		+	+
continuation of limit cycles	+	+	+
detection of branch points and codim 1 bifurcations (limit points, flip and Neimark-Sacker (torus)) of cycles	+	+	+
continuation of codim 1 bifurcations of cycles	+		+

Features of AUTO, CONTENT, and MATCONT

	A	C	M
branch switching at equilibrium and cycle bifurcations	+	+	+
continuation of branching points of equilibria and cycles			+
computation of normal forms for codim 1 bifurcations of cycles			+
detection of codim 2 bifurcations of cycles			+
continuation of orbits homoclinic to equilibria	+		

Demos: Peroxidase-oxidase reaction

- $2Y\,H_2 + O_2 + 2H^+ \rightarrow 2Y\,H^+ + 2H_2O$

$$
\begin{aligned}
A + B + X &\xrightarrow{k_1} 2X, & Y &\xrightarrow{k_5} Q,\\
2X &\xrightarrow{k_2} 2Y, & X_0 &\xrightarrow{k_0} X,\\
A + B + Y &\xrightarrow{k_3} 2X, & A_0 &\xleftrightarrow{k_7} A,\\
X &\xrightarrow{k_4} P, & B_0 &\xrightarrow{k_8} B
\end{aligned}
$$

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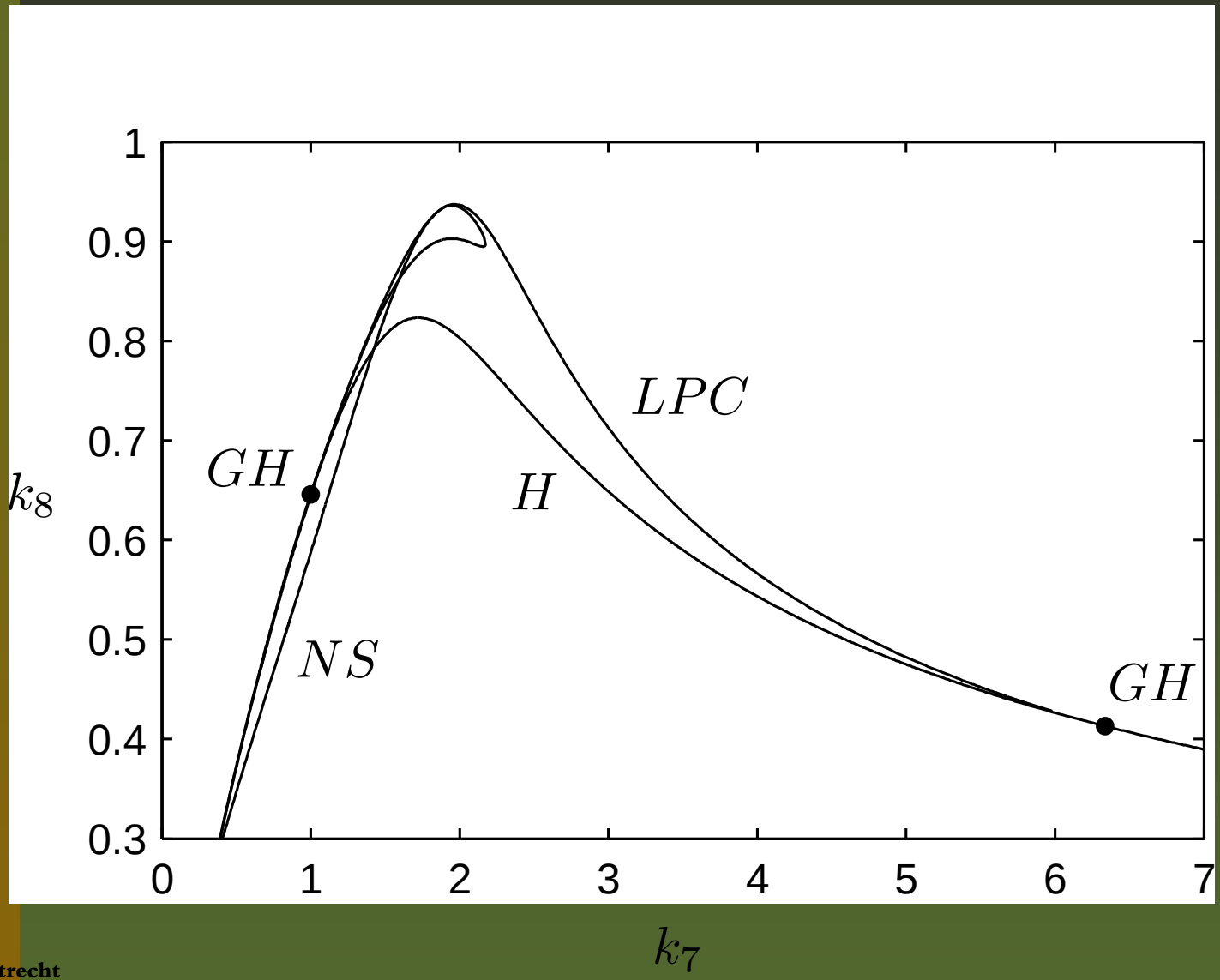
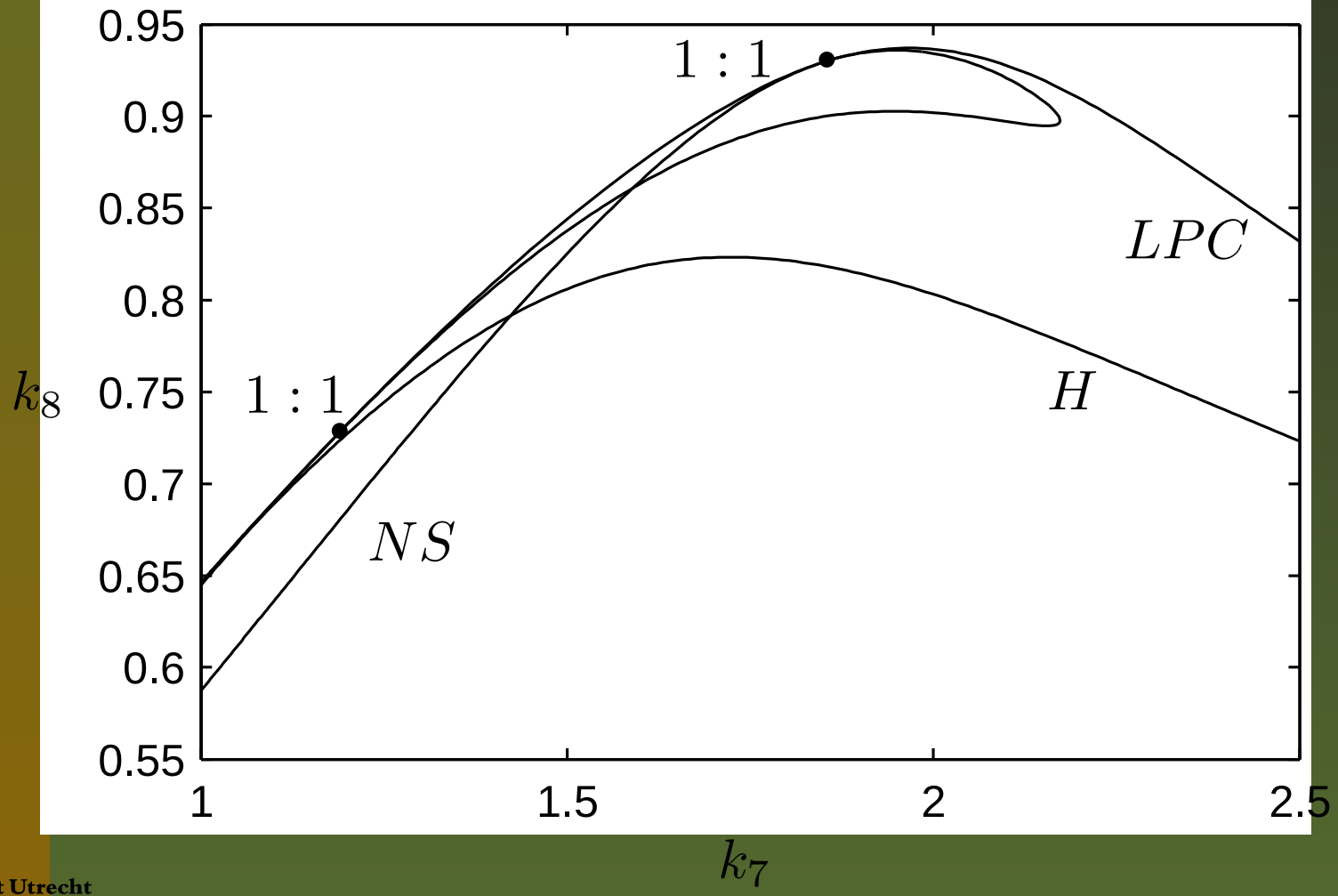
- Steinmetz & Larter [*J. Chem. Phys.* **74** (1991), 1388-1396]

$$\begin{cases} \dot{A} &= -k_1ABX - k_3ABY + k_7 - k_{-7}A, \\ \dot{B} &= -k_1ABX - k_3ABY + k_8, \\ \dot{X} &= k_1ABX - 2k_2X^2 + 2k_3ABY - k_4X + k_6, \\ \dot{Y} &= -k_3ABY + 2k_2X^2 - k_5Y. \end{cases}$$

Bifircation curves



Bifircation curves (zoom)



Perspectives

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- *Friedman, M., Govaerts, W., Kuznetsov, Yu.A., and Sautois, B.* Continuation of homoclinic orbits in MATLAB. In: V.S. Sunderam et al. (eds.) "Proceedings of the International Conference on Computational Science ICCS 2005, Atlanta, GA, USA, May 22-25, 2005, Part I". *Springer Verlag Lecture Notes in Computer Science* **3514** (2005), 263-270
- Compiling of defining functions and their Jacobian matrices