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Exploring Borders of Chaos

Prof. dr. Yuri Kuznetsov

Yuri A. Kuznetsov or Iourii A. Kouznetsov ???

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Name	Dept.	Sect.	Building	Room	Phone	Phone2
Kouznetsov, prof.dr. I.A.	EWI	AAMP	Citadel	H317	**3408**	3372

1 result found.

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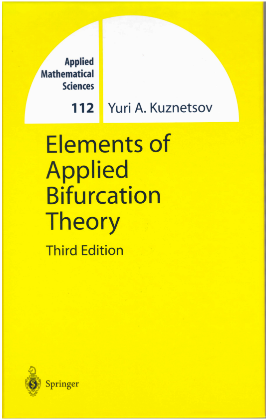

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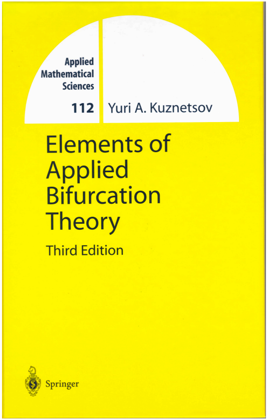
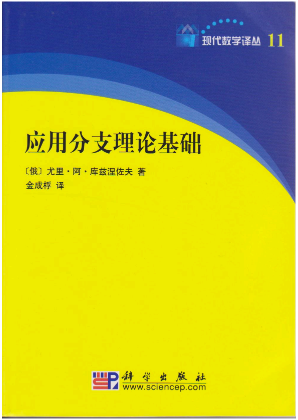
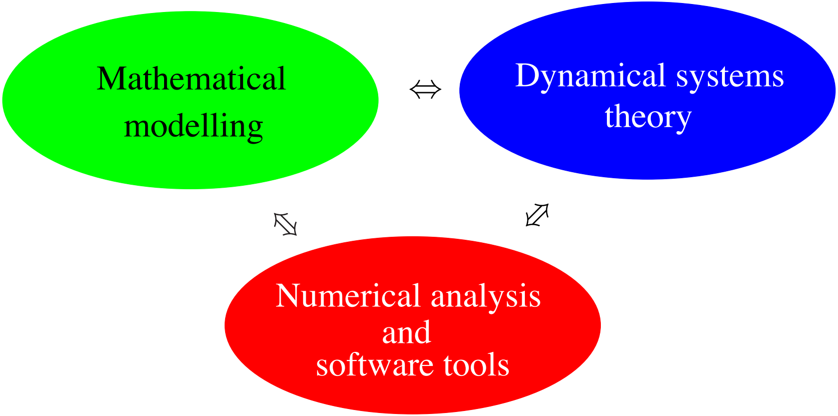




Overview

Connected research fields



P.S. de Laplace (1749-1827)

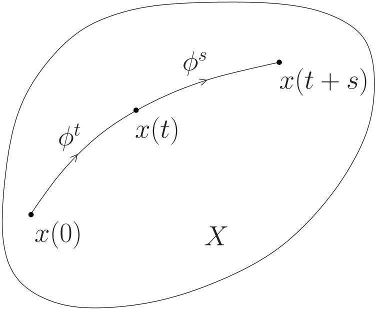
We may regard the present state of the universe as the effect of its past and the cause of its future. An intellect which at a certain moment would know all forces that set nature in motion, and all positions of all items of which nature is composed, if this intellect were also vast enough to submit these data to analysis, it would embrace in a single formula the movements of the greatest bodies of the universe and those of the tiniest atom; for such an intellect nothing would be uncertain and the future just like the past would be present before its eyes.

de Laplace, *A Philosophical Essay on Probabilities*

Dynamical systems

$$x(t) = \phi^t(x(0))$$

$$\phi^0 = \text{id}$$
$$\phi^{t+s} = \phi^t \circ \phi^s$$



Differential equations and dynamical systems

$$
\begin{cases}
\frac{dx_1(t)}{dt} & = & f_1(x_1(t), x_2(t), \ldots, x_n(t), \alpha_1, \alpha_2, \ldots, \alpha_p) \\
\frac{dx_2(t)}{dt} & = & f_2(x_1(t), x_2(t), \ldots, x_n(t), \alpha_1, \alpha_2, \ldots, \alpha_p) \\
 & \vdots & \\
\frac{dx_n(t)}{dt} & = & f_n(x_1(t), x_2(t), \ldots, x_n(t), \alpha_1, \alpha_2, \ldots, \alpha_p)
\end{cases}
$$

or

$$
\dot{x} = f(x, \alpha), \quad x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \in \mathbb{R}^n, \; \alpha = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_p \end{pmatrix} \in \mathbb{R}^p
$$

$$
\phi^t(x(0)) := x(t)
$$

Bernoulli system

$$\begin{cases} \dot{x} &= 1 \\ \dot{y} &= -x + y^2 \end{cases}$$

or

$$y' = y^2 - x$$

Bernoulli system

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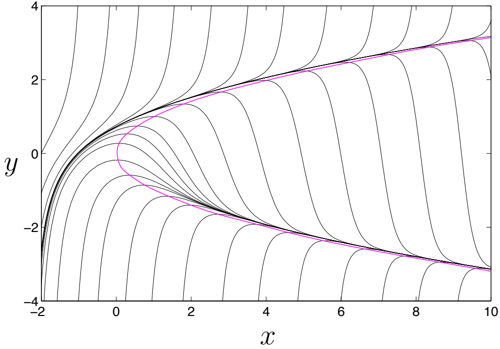
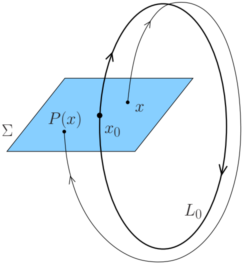


J.H. Poincaré (1854-1912)

Return map:



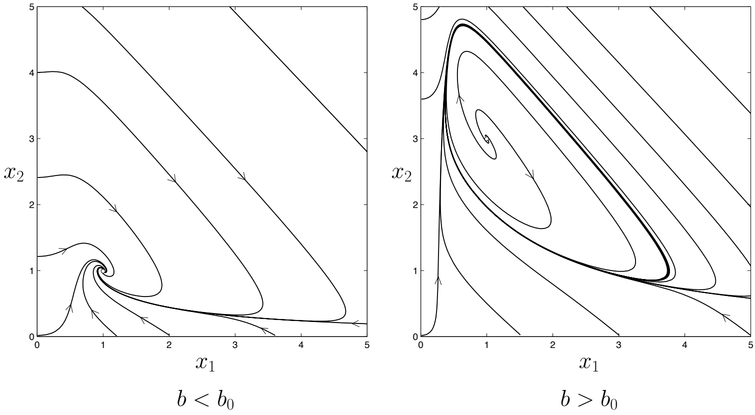
Limit cycle
Homoclinic tangle
Bifurcation

Andronov-Hopf bifurcation in Brusselator

$$\begin{cases} \dot{x}_1 & = & a-(b+1)x_1+x_1^2x_2 \\ \dot{x}_2 & = & bx_1-x_1^2x_2 \end{cases}$$

Andronov-Hopf bifurcation in Brusselator

$$\begin{cases} \dot{x}_1 = a - (b+1)x_1 + x_1^2 x_2 \\ \dot{x}_2 = b x_1 - x_1^2 x_2 \end{cases}$$



A strange attractor in the Rössler system

$$\begin{cases} \dot{x}_1 &= -x_2 - x_3 \\ \dot{x}_2 &= x_1 + Ax_2 \\ \dot{x}_3 &= Bx_1 - Cx_3 + x_1x_3 \end{cases}$$

$$A = 0.36$$
$$B = 0.4$$
$$C = 0.4$$

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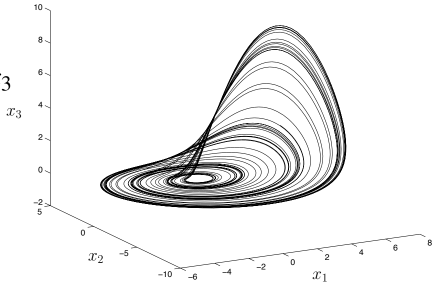
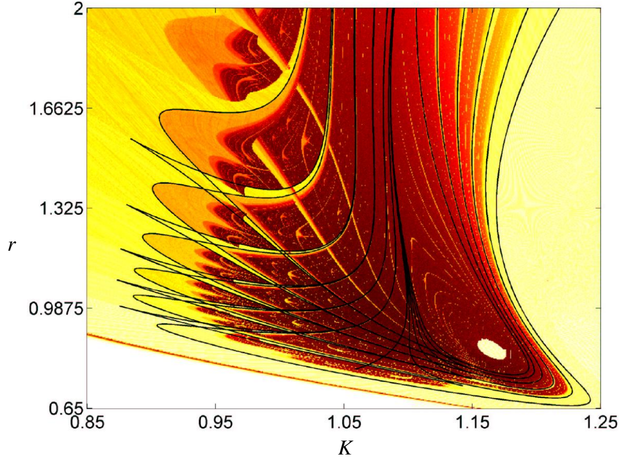


Complexity of dynamical systems

Most differential equations admit neither exact analytic solution nor a reasonably complete qualitative analysis.

V.I. Arnold, *Geometrical Methods in the Theory of Ordinary Differential Equations*

Bifurcation set of the food chain model



Kuznetsov, De Feo & Rinaldi [2001]

Overview

Normal forms for oscillatory instability

- Andronov-Hopf bifurcation:

$$\begin{cases} \dot{x}_1 &= \alpha x_1 - x_2 + l_1 x_1 (x_1^2 + x_2^2) \\ \dot{x}_2 &= x_1 + \alpha x_2 + l_1 x_2 (x_1^2 + x_2^2) \end{cases}$$

or

$$\begin{cases} \dot{\rho} &= \rho(\alpha + l_1 \rho^2) \\ \dot{\theta} &= 1 \end{cases}$$

Normal forms for oscillatory instability

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or

$$\begin{cases} \dot{\rho} &= \rho(\alpha + l_1 \rho^2) \\ \dot{\theta} &= 1 \end{cases}$$

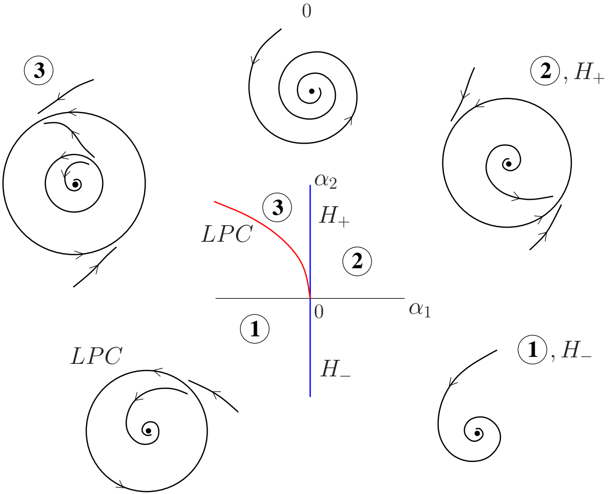
- Bautin bifurcation:

$$\begin{cases} \dot{x}_1 &= \alpha_1 x_1 - x_2 + \alpha_2 x_1 (x_1^2 + x_2^2) + l_2 x_1 (x_1^2 + x_2^2)^2 \\ \dot{x}_2 &= x_1 + \alpha_1 x_2 + \alpha_2 x_2 (x_1^2 + x_2^2) + l_2 x_2 (x_1^2 + x_2^2)^2 \end{cases}$$

or

$$\begin{cases} \dot{\rho} &= \rho(\alpha_1 + \alpha_2 \rho^2 + l_2 \rho^4) \\ \dot{\theta} &= 1 \end{cases}$$

Bautin bifurcation diagram ($l_1 < 0$)

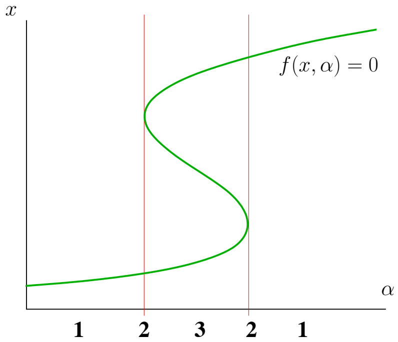


Continuation of equilibria in $\dot{x} = f(x, \alpha)$

$$F(U) = 0, \; F : \mathbb{R}^{n+1} \to \mathbb{R}^n$$

where

$$U = (x, \alpha),$$
$$F(U) = f(x, \alpha)$$



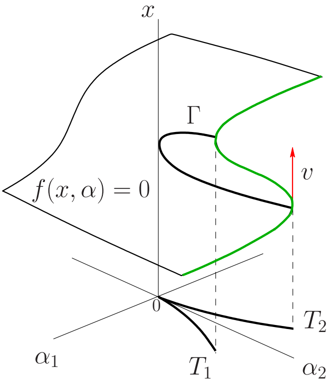
Continuation of folds

$$\begin{cases} f(x,\alpha) &= 0 \\ f_x(x,\alpha)v &= 0 \\ \langle w, v\rangle - 1 &= 0 \end{cases}$$

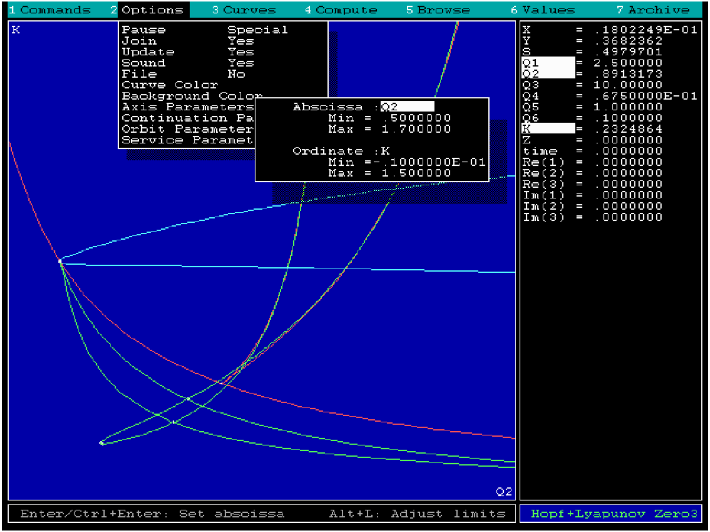
$$\begin{cases} f(x,\alpha) &= 0 \\ \det(f_x(x,\alpha)) &= 0 \end{cases}$$

$$\begin{cases} f(x,\alpha) &= 0 \\ g(x,\alpha) &= 0 \end{cases}$$ where

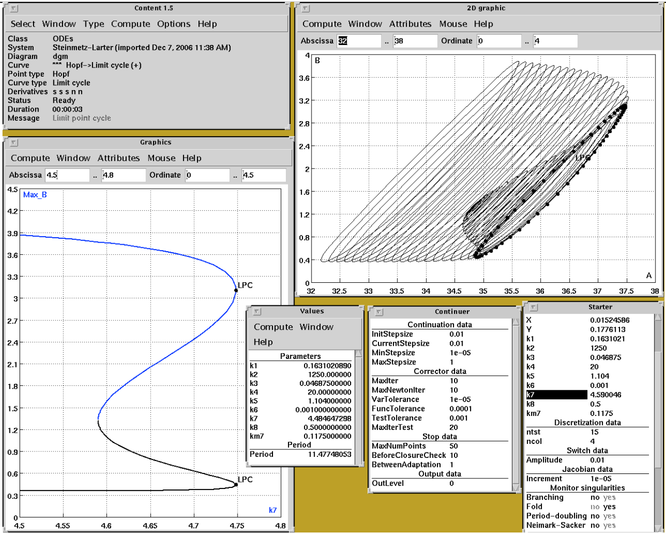
$$\begin{pmatrix} f_x(x,\alpha) & u \\ w^T & 0 \end{pmatrix} \begin{pmatrix} v \\ g \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$



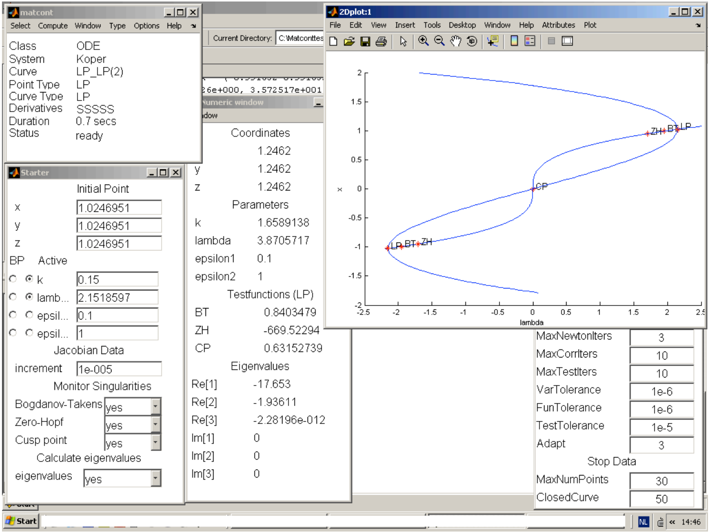
Generation I: LOCBIF (1991-1993)



Generation II: CONTENT (1993-1998)



Generation III: MATCONT (2000-)



Overview

Introduction

Numerical bifurcation analysis

Bifurcations in Neuroscience

Acknowledgements

Double impulses in FitzHugh-Nagumo model

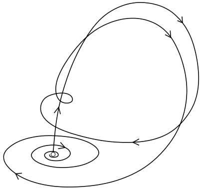
$$\left\{\begin{array}{rcl}\frac{\partial V}{\partial t} & = & \frac{\partial^2 V}{\partial x^2} - f(V) - W \\ \frac{\partial W}{\partial t} & = & b(V - \gamma W)\end{array}\right. \Rightarrow \left\{\begin{array}{rcl}\frac{dv}{d\xi} & = & u \\ \frac{du}{d\xi} & = & cu + f(v) + w \\ \frac{dw}{d\xi} & = & \frac{b}{c}u(v - \gamma w)\end{array}\right.$$

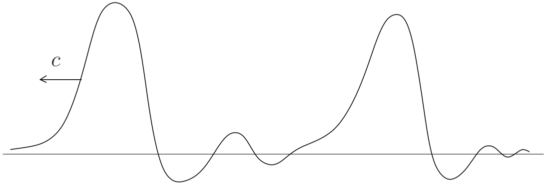
$$V(t,x) = v(\xi),\ W(t,x) = w(\xi),\ \xi = x + ct$$

Double impulses in FitzHugh-Nagumo model

$$\begin{cases} \dfrac{\partial V}{\partial t} = \dfrac{\partial^2 V}{\partial x^2} - f(V) - W \\ \dfrac{\partial W}{\partial t} = b(V - \gamma W) \end{cases} \Rightarrow \begin{cases} \dfrac{dv}{d\xi} = u \\ \dfrac{du}{d\xi} = cu + f(v) + w \\ \dfrac{dw}{d\xi} = \dfrac{b}{c} u(v - \gamma w) \end{cases}$$

$$V(t,x) = v(\xi),\ W(t,x) = w(\xi),\ \ \xi = x + ct$$





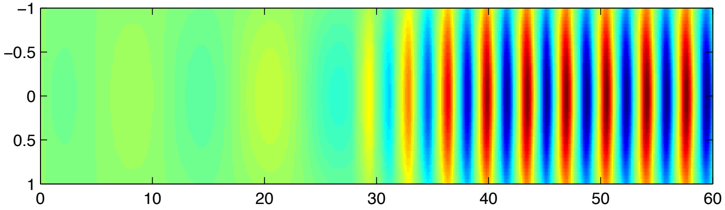
Bifurcations of neural field models

$$\frac{\partial V(t,x)}{\partial t} = -\alpha V(t,x) + \int_{\Omega} w(x,x') f\left(V\left(t - \tau_0 - \frac{|x-x'|}{c}, x'\right)\right) dx'$$

Bifurcations of neural field models

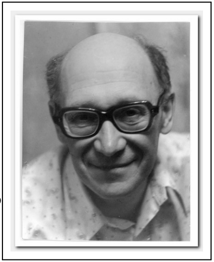
$$\frac{\partial V(t,x)}{\partial t} = -\alpha V(t,x) + \int_{\Omega} w(x,x') f\left(V\left(t - \tau_0 - \frac{|x-x'|}{c}, x'\right)\right) dx'$$

Andronov-Hopf bifurcation:



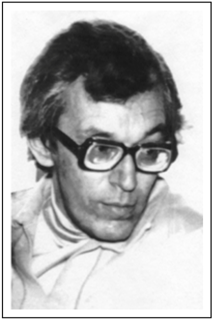
Overview

My teachers at the RCC (Pushchino)



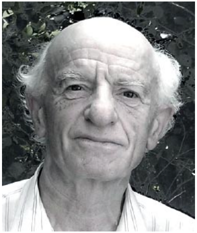
A.M. Molchanov

(1928-2011)



A.D. Bazykin

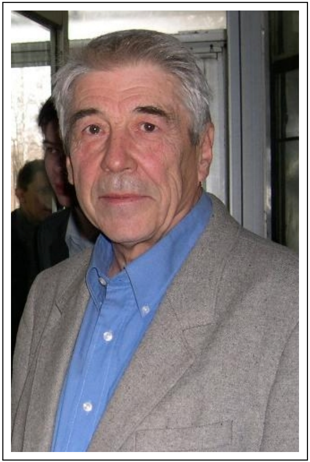
(1940-1994)



E.E. Shnol

(1928-)

V.I. Arnold (1937-2010)



L.P. Shilnikov (1934-2011)

Supervised PhD Thesis