Practicum 1, 18 August 2009, 15:15-17:00 uur

For each planar system below, construct its phase portrait numerically using the `MATLAB` tool `pplane7`[1] and then try to prove its essential features analytically.

- **Lotka-Volterra system**

$$\left\{ \begin{array}{rcl} \dot{x} & = & x - xy, \\ \dot{y} & = & -y + xy, \end{array} \right. \tag{1}$$

  where $x, y \geq 0$.

  Hint: Introduce new variables $q = \ln x$ and $p = \ln y$ and prove that the resulting (q, p)-system is Hamiltonian.

- **A system without cycles**

$$\left\{ \begin{array}{rcl} \dot{x} & = & y, \\ \dot{y} & = & -x - y + x^2. \end{array} \right. \tag{2}$$

- **Reversible system**

$$\left\{ \begin{array}{rcl} \dot{x} & = & y, \\ \dot{y} & = & x + xy - x^3. \end{array} \right. \tag{3}$$

  Hint: Consider the transformation $(x, y, t) \to (-x, y, -t)$.

- **A system with a nonsimple equilibrium**

$$\left\{ \begin{array}{rcl} \dot{x} & = & x^2 - y^2, \\ \dot{y} & = & 2xy. \end{array} \right. \tag{4}$$

  Hint: The system is equivalent to one complex equation $\dot{z} = z^2$.

- **A system with a saddle homoclinic orbit**

$$\left\{ \begin{array}{rcl} \dot{x} & = & -x + 2y + x^2, \\ \dot{y} & = & 2x - y - 3x^2 + \frac{3}{2}xy. \end{array} \right. \tag{5}$$

  Hint: The curve $x^2(1 - x) - y^2 = 0$ is invariant.

[1] To use the tool, start `MATLAB` by selecting **Start** → **All Programs** → **MATLAB** → **R2009a** → **MATLAB R2009a** and enter `pplane7` in the `MATLAB` Command Window.