Seminar Constructible Sets: Handout 10

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Definition 1. A subset E of a limit ordinal α is said to be *stationary* in λ iff E has a non-empty intersection with every club subset of λ.

Definition 2. Let λ be an ordinal and $E \subseteq \lambda$. A function $f : E \to \lambda$ is *regressive* if for every non-zero $\alpha \in E$, $f(\alpha) < \alpha$.

Lemma 3 (Homework). *Given an uncountable regular cardinal λ, a set $E \subseteq \lambda$ is stationary in λ iff every reductive function $E \to \lambda$ is constant on some unbounded subset of E.*

Definition 4. The *diamond principle* $\diamond$ is the statement

> There is a sequence $(S_\alpha \mid \alpha < \omega_1)$ such that $S_\alpha \subseteq \alpha$, with the property that whenever $X \subseteq \omega_1$, the set $\{\alpha \in \omega_1 \mid X \cap \alpha = S_\alpha\}$ is stationary in ω_1.

Theorem 5 (Devlin III 3.2). *$\diamond$ implies the existence of a Souslin tree.*