Seminar H10: exercises week 2

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Due 7 October 2013

Exercise 1

For natural numbers k, let S_k be the sequence of digits $k(k-1)...10$. Give an exponential Diophantine equation $E_L(a, b) = E_R(a, b)$ such that we have $\forall k \exists x E_L(x, k) = E_R(x, k)$ and $\forall x \forall k (E_L(x, k) = E_R(x, k) \rightarrow \exists b(\widetilde{x}(b) = S_k))$, where $\widetilde{x}(b)$ denotes the digit representation of x relative to base b. Does this yield a exponential Diophantine representation of the relation $R(x, k) :\Leftrightarrow \exists b(\widetilde{x}(b) = S_k)$?

Exercise 2

Let $m(x) = k$ express that x masks exactly k numbers.
a) Give an exponential Diophantine representation of the property $m(x) = 2$.
b) Let b and c be natural numbers such that $b \preceq c$. Give a formula which expresses $m(c - b)$ in terms of $m(c)$, $m(b)$ and $m(b \wedge c)$.
c) Can you give a similar formula for arbitrary b and c (i.e., b and c for which the condition $b \preceq c$ does not necessarily hold)?