Tame Topology and O-minimal Structures,
Euler Characteristic, homework set
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We take an o-minimal structure $(R, <, S)$.

1 Cell decomposition (5 points)

Take a cell $C \subset R^m$. This exercise tackles the similarity between the definition of a cell decomposition of R^m and the definition of a decomposition of a cell. The definition of a decomposition of a cell is given on page 70.

a. (2 points) Prove that that if $\mathbf{D}$ is a cell decomposition of R^m that partitions C, than $\mathbf{D}|C = \{E : E \epsilon \mathbf{D}, E \subseteq C\}$ is a decomposition of C.

b. (3 points) Prove that for any decomposition $\mathbf{D}$ of C, there is a cell decomposition of R^m that restricts to $\mathbf{D}$ on C.

2 Closure (5 points)

Prove that the Euler characteristic of the closure of a bounded cell $C \subset R^m$ is always 1. Bounded means there is a box $B = [a_0, b_0] \times [a_1, b_1] \times ... \times [a_n, b_n]$ with $a_i, b_i \epsilon R$ for all i, such that $C \subset B$.

Hint: Use induction and consider the cases $i_m = 0$ and $i_m = 1$ separately. Use proposition 2.4.