INTAS PROPOSAL FOR
Open call 2005 - Full Proposal - Research Project

1.1 TITLE:
Singularities, Bifurcations and Monodromy

1.1.1 Keyword 1 : Geometry, Algebraic Geometry
Keyword 2 : Topology & Manifolds
Keyword 3 : Dynamical Systems (including Ergodic Theory, Fuzzy, Chaotic Systems etc.)

1.1.2 Free word 1 : singularities
Free word 2 : bifurcations
Free word 3 : monodromy

1.1.3 Intended Start Date: July 2006

1.1.4 Duration: 24 Months

1.2 CONSORTIUM

Universiteit Utrecht - The Netherlands
Hannover University - Germany
Institut de Mathématiques de Luminy - France
Universidad de Valladolid - Spain
Institute of Mathematics - Poland
University of Liverpool - United Kingdom
Independent University of Moscow - Russia
Moscow State University - Russia
Steklov Mathematical Institute - Russia
Moscow Aviation Institute - Russia
A.Razmadze Mathematical Institute - Georgia

1.3 SUMMARY

The aim of the project is a wide range of research in the Singularity Theory, study of bifurcations and monodromy. These fields are closely adjacent to a number of various problems of calculus, algebraic geometry, topology, ... During last decades Singularity Theory benefited a lot from using methods of calculus, algebraic geometry, topology, and also influenced researches in these traditional fields of mathematics. Singularity Theory and its applications traditionally developed within two-way influence and cooperation between the West European and fSU groups of researches. In this an important role was played by the European Singularity Network (now terminated) and by three INTAS Projects: Investigations in singularity theory (94-4373, 1995-1997) and Topology and analysis of discriminant sets (96-0713, 1997-1999), Singularity Theory and Bifurcations (00-0259, 2001-2003). All three INTAS Projects were coordinated by the coordinator of this Project Prof.Dr. D.Siersma. He also played an important role in the coordination of the European Singularity Network. In frames of those Projects and of the Network there were elaborated a number of problems in Singularity Theory and in Bifurcation Theory, there were obtained a number of new results (more than 300 papers were published or accepted for publication in framework of the INTAS Projects; many of them are joint papers of fSU and Western participants, prepared in the framework of the Projects) and there was created a good scientific environment for further cooperation between West European and fSU groups of

researchers. The groups involved in the Project include the majority of active researchers in the Singularity Theory from the fSU and from West and Central Europe. The Project is supposed to considerably extend the knowledge in the area by further developing existing problems and by studying a number of new ones on the base of cooperation between the groups.
The Project brings together 5 research teams from the fSU: Independent Univ. of Moscow, Mathematics Colledge (Russia, Moscow), 6 members, team leader V.A.Vassiliev; Moscow State Univ. (Russia), Faculty of Mechanics and Mathematics, 7 members, team leader S.M.Gusein-Zade; Moscow Aviation Inst. (Russia), 8 members, team leader V.M.Zakalyukin; Steklov Mathematical Inst. (Russia, Moscow), 5 members, team leader Vik.S.Kulikov; Georgian Academy of Sciences, A.Razmadze Mathematical Inst. (Georgia), 5 members, team leader G.N.Khimshiashvili; - and 6 research teams from INTAS countries: The Netherlands, Univ. Utrecht, Mathematisch Inst., 9 members, team leader, the Coordinator of the Project D.Siersma; Germany, Univ. of Hannover, 9 members, team leader W.Ebeling; France, Marseille-Nice-Strasbourg Univ., 9 members, team leader J.P. Brasselet; Great Britain, Univ. of Liverpool, 9 members, team leader V.Goryunov; Spain, Univ. of Valladolid, 12 members, team leader A.Campillo; Poland, Inst. of Mathematics, Warsaw, 8 members, team leader S.Janeczko.

2 TEAM INFORMATION

2.1 Team : Utrecht University

2.1.1 Team Description

The team is based on the Department of Mathematics of the University of Utrecht. It contains also members from the universities of Nijmegen and Groningen.
The team has expertise in:
- topology and geometry of singularites (including non-isolated singularities)
- geometry of moduli spaces
- 3-manifolds
- conflict sets, dynamics of wave fronts
- mathematical physics

More in detail:
D. Siersma: non-isolated singularities, topology and geometry of conflict sets
J. SteenbrinKk: mixed Hodge structures, singularities, moduli spaces
J. Stienstra: toric geoemtry and hypergeometric systems
E. Looijenga: Algebraic and analytic geometry, in particular, moduli spaces.
L. Hoevenaars: Special Kaehler varieties, integrable systems, WDVV- equations
S. Anisov: 3-manifolds, conflict sets, topology and geometry of singularities
J. Duistermaat: partial differential equations, Hamiltaonian systems, Lie theory.
H. Broer: dynamical system theory, including: bifurcation theory, applied singularity theory
G. Vegter: Computational Geometry, Singularity Theory and its Applications.

The team has direct relations with almost all participating institutes and complements their tasks. The infrastructure is the equipment of the Mathematical Research Institute (MRI) in The Netherlands, which includes international Master Classes and profits from Netherlands-Russian project on Singularities and Mathematical Physics.

2.1.2 List of publications

1 D. Siersma, M.Tibar: Deformations of polynomials, boundary singularities and monodromy;
Mosc. Math. J. 3 (2003), no. 2, 661--679, 745.
2 C. Peters, J. Steenbrink: Degeneration of the Leray spectral sequence for certain geometric quotients: Moscow Mathematical Journal 3 (3) 2003, 1085-1095
3 W.Couwenberg, G.Heckman, E.Looijenga:
Geometric structures on the complement of a projective arrangement;
Publ. Math., Inst. Hautes Étud. Sci. 101, 69-161 (2005).
4 D. Siersma, C.T.C. Wall and V. Zakalyukin (editors):
New Developments in Singularity Theory. Proceedings of the NATO Advanced Study Institute held at Cambridge, July 31-Augusts 11, 2000. Nato Science series II: Mathematics, Physics and Chemistry, 21. Kluwer Academic Publishers, Dordrecht, 2001.

2.1.3 Team Leader and address

Title	Prof.
Position	Scientific Director
Sex	Male
Date Of Birth	25/08/1943

First Name	Dirk
Patronic Name	
Family Name	Siersma
Organisation Type	Public
Organisation Registration Nr	
Academy / Branch	
Organisation / University / Institute	Universiteit Utrecht
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Laboratory	Mathematical Research Institute
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2.1.4 List of Senior Scientists in the team

1)	First Name	Sergei
	Patronic Name	Semenovich
	Family Name	Anisov
	Year Of Birth	1972
	Insitute	Mathematisch Instituut Universiteit Utrecht
2)	First Name	Hendrik
	Patronic Name	
	Family Name	Broer
	Year Of Birth	1950
	Insitute	Instituut voor Wiskunde en Informatica Rijksuniversiteit Groningen
3)	First Name	Johannes
	Patronic Name	
	Family Name	Duistermaat
	Year Of Birth	1942
	Insitute	Mathematisch Instituut Universiteit Utrecht

4) First Name	Luuk
Patronic Name	
Family Name	Hoevernaars
Year Of Birth	1975
Insitute	Mathematisch Instituut Universiteit Utrecht
5) First Name	Eduard
Patronic Name	
Family Name	Looijenga
Year Of Birth	1948
Insitute	Mathematisch Instituut Universiteit Utrecht
6) First Name	Joseph
Patronic Name	
Family Name	Steenbrink
Year Of Birth	1947
Insitute	Mathematisch Instituut Radbout Universiteit Nijmegen
7) First Name	Jan
Patronic Name	
Family Name	Stienstra
Year Of Birth	1950
Insitute	Mathematisch Instituut Universiteit Utrecht
8) First Name	Gert
Patronic Name	
Family Name	Vegter
Year Of Birth	1954
Insitute	Instituut voor Wiskunde en Informatica Rijksuniversiteit Groningen

2.1.5 Statistics

Number of Team Members involved in this project: 9
Number of Team Members under 35: 2
Number of Team Members who have individually received grants in INTAS projects: 0

2.2 Team : Hannover University

2.2.1 Team Description

W. Ebeling: Vector fields, 1-forms and collection of 1-forms on singular varieties, monodromy of singularities
A. Frühbis-Krüger: Classification of singularities, computational aspects of resolutions, moduli spaces
G.-M. Greuel: Representation theory and singularities, Computations in non commutative algebras
H. Hamm: Geometry of non-proper mappings of complex algebraic varieties, equisingularity, torus compactifications, De Rham cohomology, Hodge theory

C. Hertling: Gauss-Manin connection of complete intersection singularities
G. Pfister: Computer algebra, Representation theory and singularities
A. Pratoussevitch: Moduli spaces of quasihomogeneous Gorenstein surface singularities
J. Schürmann: Geometry of singular spaces, characteristic classes, stratified Morse theory, constructible and perverse sheaves
D. van Straten: Symplectic singularities

2.2.2 List of publications

1 Indices of vector fields or 1-forms and characteristic numbers, W.Ebeling, S.M.Gusein-Zade, Bull. London Math. Soc. 37 (2005), 747-754.

2 On Cohen-Macaulay modules on surface singularities, Y.Drozd, G.-M.Greuel, I.Kashuba,
Mosc. Math. J. 3 (2003), 397-418, 742.
3 Frobenius manifolds and moduli spaces for singularities, C.Hertling, Cambridge Tracts in Math. 151. Cambridge University Press, Cambridge, 2002.
4 The topology of m-spinor structures on Riemann surfaces, S.M.Natanzon, A.M.Pratoussevitch, Uspekhi Mat. Nauk 60 (2005), 169-170; Engl. translation in Russian Math. Surveys 60 (2005), 363-364.
5 Deformation of singular Lagrangian subvarieties, Ch.Sevenheck, D.van Straten, Math. Ann. 327 (2003), 79-102.

2.2.3 Team Leader and address

Title	Prof.
Position	Staff Member
Sex	Male
Date Of Birth	05/10/1951
First Name	Wolfgang
Patronic Name	
Family Name	Ebeling
Organisation Type	Public
Organisation Registration Nr	
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Website

2.2.4 List of Senior Scientists in the team

1) First Name Anna
   Patronic Name
   Family Name Frühbis-Krüger
   Year Of Birth 1970
   Insitute Kaiserslautern Technical University
2) First Name Gert-Martin
   Patronic Name
   Family Name Greuel
   Year Of Birth 1944
   Insitute Kaiserslautern Technical University
3) First Name Helmut
   Patronic Name
   Family Name Hamm
   Year Of Birth 1944
   Insitute Münster University
4) First Name Claus
   Patronic Name
   Family Name Hertling
   Year Of Birth 1965
   Insitute Mannheim University
5) First Name Gerhard
   Patronic Name
   Family Name Pfister
   Year Of Birth 1947
   Insitute Kaiserslautern Technical University
6) First Name Anna
   Patronic Name
   Family Name Pratoussevitch
   Year Of Birth 1974
   Insitute Bonn University
7) First Name Jörg
   Patronic Name
   Family Name Schürmann
   Year Of Birth 1963
   Insitute Münster University
8) First Name Duco
   Patronic Name

Family Name	van Straten
Year Of Birth	1958
Insitute	Mainz University

2.2.5 Statistics

Number of Team Members involved in this project: 9
Number of Team Members under 35: 1
Number of Team Members who have individually received grants in INTAS projects: 0

2.3 Team : Marseille-Nice-Stra.

2.3.1 Team Description

The main research areas of the team, in connection with the INTAS project are the following:
Jean-Paul Brasselet:
- Stratifications of real and complex algebraic/analytic varieties,
Whitney and Thom stratifications.
- Polar varieties and characteristic classes,
- Stratified vector fields and characteristic classes of singular spaces:
Schwartz-MacPherson, Fulton, Mather, Milnor classes...
- Toric varieties (homology, characteristic classes),
- Intersection homology and cyclic homology.

Nicolas Dutertre and David Trotman:
- Topological classification of complex singularities and invariance of multiplicity.
- Equisingularity theory (spaces and functions),
- Link between Zariski equisingularity and Lipschitz equisingularity,
- Stratifications of real and complex algebraic/analytic varieties and their projections
(semialgebraic / subanalytic sets).
- Resolution of singularities and the local geometric structure of singular spaces.
Applications to classification and stability problems.
- Metric classification (bilipschitz structures),
- Stratified vector fields and isotopies. Applications to homology/cohomology of singular spaces
and maps.

Viatcheslav Kharlamov, Nermine Salepci and Vladimir Turaev:
- Deformations of real and complex varieties.
- Deformations and topological invariants of real and complex mappings.
- real Lefschetz sheaves,

Philippe Maisonobe and Michel Merle:
- Whitney and Thom stratifications.
- Polar varieties and Chern-Mather classes,
- Motivic integration, motivic Milnor fibre, Convolution and composition with a function (Thom-Sebastiani),
- D-modules on a complex manifold. Topological aspects via Riemann-Hilbert with perverse sheaves, vanishing cycles, Mellin transform for a perverse sheaf,
- real and complex deformation theories,

Anne Pichon:
- Topology of singularities of complex spaces, of complex analytic germs,

- Topology of real analytic germs,
- Non isolated singularities (of surfaces),
- Knot theory applied to singularities,

2.3.2 List of publications

1 On real structures on rigid surfaces, S. Kharlamov and V. Kulikov, published in Izvestiya
2 Real "Dif=Def" problem", S. Kharlamov and V. Kulikov, submitted to Izvestiya,
3 Stratifications and finite determinacy, D.Trotman & L. Wilson, Proceedingsof the London Math. Soc. (3) 78, 1999, 334-368.
4 Euler obstruction and defects of functions on singular varieties, J.P. Brasselet, D. Mond, A.J. Parameswaran and J. Seade, Journal London Soc. Math. (2) 70 (2004) 59-76.
5 A compactification of the Hurwitz space,
Sergei Natanzon and Vladimir Turaev,
Topology 38 (1999), no. 4, 889--914.

2.3.3 Team Leader and address

Title	Prof.
Position	Research Director
Sex	Male
Date Of Birth	22/03/1945
First Name	Jean-Paul
Patronic Name	
Family Name	Brasselet
Organisation Type	Public
Organisation Registration Nr	
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2.3.4 List of Senior Scientists in the team

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   Family Name Dutertre
   Year Of Birth 1971
   Insitute Université de Provence
2) First Name Viatcheslav
   Patronic Name
   Family Name Kharlamov
   Year Of Birth 1950
   Insitute Université de Strasbourg
3) First Name Philippe
   Patronic Name
   Family Name Maisonobe
   Year Of Birth 1948
   Insitute Université de Nice
4) First Name Michel
   Patronic Name
   Family Name Merle
   Year Of Birth 1948
   Insitute Université de Nice
5) First Name Anne
   Patronic Name
   Family Name Pichon
   Year Of Birth 1968
   Insitute Université de la Méditerranée
6) First Name Nermine
   Patronic Name
   Family Name Salepci
   Year Of Birth 1976
   Insitute Université de Strasbourg
7) First Name David
   Patronic Name
   Family Name Trotman
   Year Of Birth 1951
   Insitute Université de Provence
8) First Name Vladimir
   Patronic Name
   Family Name Turaev
   Year Of Birth 1954
   Insitute Université de Strasbourg

2.3.5 Statistics

Number of Team Members involved in this project: 9
Number of Team Members under 35: 2
Number of Team Members who have individually received grants in INTAS projects: 0

2.4 Team : Valladolid Univ.

2.4.1 Team Description

Main research areas of the team, in connection with the INTAS project,
A.Campillo. Classification of singularities, Poincare series, curve singularities.
F.Delgado de la Mata. Equisingularity, singularities of critical sets, Poincare series.
I. Luengo. Singularity invariants, monodromy, motivic integration.
A. Melle-Hernandez. Singularity invariants, monodromy, motivic integration.
J. Fernandez de Bobadilla. Monodromy, automorphism
groups.
E.Bujalance. Riemmann and Klein surfaces, automorphism groups.
A. Costa. Topology of varieties, hyperbolic geometry.
A.M. Porto. Riemmann and Klein surfaces, covering spaces.
A. Lemahieu. Monodromy, Poincare series.
F. Hernando. Critical sets singularities, singular 1-forms.
H. Cobo. Monodromy, motivic integration.
M. Gonzalez Villa. Monodromy, motivic integration.

2.4.2 List of publications

1 The Alexander polynomial of a plane curve singularity and the ring of functions on it. A.Campillo, F.Delgado, S.Gusein-Zade. Duke Math. J. 117(1),125-156 (2003).
2 Poincaré series of rational surface singularities. A.Campillo, F.Delgado, S.Gusein-Zade. Invent. Math. 155, 41-53 (2004).

3 A power structure over the Grothendieck ring of varieties. S.M.Gusein-Zade, I.Luengo, A.Melle-Hernandez. Math. Res. Lett, 11, 49-57 (2004).
4 Partial resolutions and the zeta-function of a singularity. S.M.Gusein-Zade, I.Luengo, A.Melle-Hernandez. Comm.Math.Helv. 72, 2, 244-256 (1997).

5 Topological classification of actions on surfaces. Antonio F. Costa, Sergei M. Natanzon. Michigan Math. J. 50, 3, 451-460 (2002).

2.4.3 Team Leader and address

Title	Prof.
Position	Group Leader
Sex	Male
Date Of Birth	26/11/1953
First Name	Antonio
Patronic Name	
Family Name	Campillo
Organisation Type	Public
Organisation Registration Nr	

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2.4.4 List of Senior Scientists in the team

1) First Name Emilio
   Patronic Name
   Family Name Bujalance
   Year Of Birth 1953
   Insitute UNED, Madrid
2) First Name Helena
   Patronic Name
   Family Name Cobo
   Year Of Birth 1980
   Insitute Universidad Complutense Madrid
3) First Name Antonio
   Patronic Name
   Family Name Costa
   Year Of Birth 1955
   Insitute UNED, Madrid
4) First Name Félix
   Patronic Name
   Family Name Delgado de la Mata
   Year Of Birth 1960
   Insitute Universidad de Valladolid
5) First Name Javier
   Patronic Name
   Family Name Fernández de Bobadilla

	Year Of Birth	1974
	Insitute	UNED, Madrid
6)	First Name	Manuel
	Patronic Name	
	Family Name	González-Villa
	Year Of Birth	1979
	Insitute	Universidad Complutense Madrid
7)	First Name	Fernando
	Patronic Name	
	Family Name	Hernando
	Year Of Birth	1976
	Insitute	Universidad de Valladolid
8)	First Name	Ann
	Patronic Name	
	Family Name	Lemahieu
	Year Of Birth	1980
	Insitute	Universidad de Valladolid, Leuven University
9)	First Name	Ignacio
	Patronic Name	
	Family Name	Luengo
	Year Of Birth	1953
	Insitute	Universidad Complutense, Madrid
10)	First Name	Alejandro
	Patronic Name	
	Family Name	Melle-Hernández
	Year Of Birth	1970
	Insitute	Universidad Complutense, Madrid
11)	First Name	Ana María
	Patronic Name	
	Family Name	Porto
	Year Of Birth	1960
	Insitute	UNED, Madrid

2.4.5 Statistics

Number of Team Members involved in this project: 12
Number of Team Members under 35: 5
Number of Team Members who have individually received grants in INTAS projects: 0

2.5 Team : Polish Ac. Sci.

2.5.1 Team Description

Areas of expertise: Algebraic geometry, Differential geometry, Real analytic and algebraic geometry, Commutative algebra, Differential topology, Partial differential equations.
The proposed team:
Stanislaw Janeczko (team leader): Symplectic geometry, singularities and

bifurcations,
Tadeusz Mostowski: metric properties of analytic sets; Lipschitz equisingularity,
Zbigniew Jelonek:geometry of polynomial mapping; images of polynomial mappings,
Wojciech Domitrz: symplectic geometry, differential forms,
Zbigniew Szafraniec: topological invariants of real algebraic sets,
Grzegorz Gromadzki: teoria powierzchni Riemanna, odwzorowania powierzchni zwartych,
Mariusz Zajac: real plane algebraic curves, singularities,
Hassan Babiker: Singularities of smooth mappings, symplectic geometry

2.5.2 List of publications

1 Z. Jelonek, "On the effective Nullstellensatz", Invent. Math. 162, 1-17, 2005.

2 W. Domitrz, S. Janeczko, M. Zhitomirskii, "Relative Poincare Lemma, contractibility, quasi-homogeneity and vector fields tangent to a singular variety" Illinois J. of Mathematics, 48, No. 3, (2004),803-835.
3 Z. Szafraniec, "Topological invariants of real Milnor fibres". Manuscripta Math. 110 (2003) 2, 159-169.
4 S. Janeczko, Z. Jelonek, "Linear authomorphisms that are symplectomorphisms", J. London Math. Soc. (2) 69 (2004) 503-517.
5 G. Ishikawa, S. Janeczko, "Symplectic bifurcations of plane curves and isotropic liftings", Quart. J. Math. 54 (2003),1-30,

2.5.3 Team Leader and address

Title	Prof.
Position	Director
Sex	Male
Date Of Birth	25/08/1953
First Name	Stanisław
Patronic Name	Tadeusz
Family Name	Janeczko
Organisation Type	Public
Organisation Registration Nr	
Academy / Branch	Polish Academy of Sciences
Organisation / University / Institute	Institute of Mathematics
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2.5.4 List of Senior Scientists in the team

1) First Name Wojciech
   Patronic Name
   Family Name Domitrz
   Year Of Birth 1967
   Insitute Warsaw Univ. of Technology
2) First Name Grzegorz
   Patronic Name
   Family Name Gromadzki
   Year Of Birth 1958
   Insitute Institute of Mathematics, Gdansk University
3) First Name Zbigniew
   Patronic Name
   Family Name Jelonek
   Year Of Birth 1962
   Insitute Polish Acad. of Sci.
4) First Name Tadeusz
   Patronic Name
   Family Name Mostowski
   Year Of Birth 1947
   Insitute Warsaw Universty
5) First Name Zbigniew
   Patronic Name
   Family Name Szafraniec
   Year Of Birth 1955
   Insitute Gdansk University
6) First Name Mariusz
   Patronic Name
   Family Name Zając
   Year Of Birth 1974
   Insitute Faculty of Math. and Inf. Sci. Warsaw Univ. of Technology

2.5.5 Statistics

Number of Team Members involved in this project: 8
Number of Team Members under 35: 2

Number of Team Members who have individually received grants in INTAS projects: 0

2.6 Team : Univ. of Liverpool

2.6.1 Team Description

J.W.Bruce: classificational problems of sigularity theory, singularities in defferential geometry
P.Giblin: singularities in differential geometry and computer vision
I.de Gregorio: Frobenius structures defined by singularities
V.Goryunov: Largangian and Legendrian singularities, monodromy of equivariant functions
K.Houston: vanishing topology of map-germs, equisingularity of maps
D.Mond: free divisors, composed maps
F.Tari: singularities in differential geometry
There are 2 PhD students in the team, D.Davis (with P.Giblin) and D.Littlestone (with K.Houston). Both are working on problems from their supervisors' areas.

2.6.2 List of publications

1 Singularities of centre symmetry sets. P.Giblin and V.Zakalyukin, Proceedings of London Mathematical Society 90(2005), 132-166, http://www.liv.ac.uk/~pjgiblin/papers/giblin-zakalyukin-accepted-single.pdf
2 Simple Symmetric Matrix Singularities and the Subgroups of Weyl Groups Am, Dm, Em. V.Goryunov and V.Zakalyukin, Moscow Math. J. 3 (2003), no.2, 507-530, http://www.liv.ac.uk/~goryunov/papers/gzfin.ps
3 Tjurina and Milnor numbers of matrix singularities. V.Goryunov and D.Mond, Journal of London Mathematical Society 72(2005), 205-224, http://www.maths.warwick.ac.uk/~mond/Papers/tm.ps
4 On families of square matrices. J.W.Bruce and F.Tari, Proceeding of London Mathematical Society 89 (2004), no.3, 738-762
5 Dupin indicatrices and families of curve congruences. J.W.Bruce, F.Tari, Trans. Amer. Math. Soc. 357 (2005), no. 1, 267--285

2.6.3 Team Leader and address

Title	Prof.
Position	Group Leader
Sex	Male
Date Of Birth	17/05/1955
First Name	Victor
Patronic Name	Vladimirovich
Family Name	Goryunov
Organisation Type	Public
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2.6.4 List of Senior Scientists in the team

1) First Name James William
   Patronic Name
   Family Name Bruce
   Year Of Birth 1952
   Insitute University of Hull
2) First Name Declan
   Patronic Name
   Family Name Davis
   Year Of Birth 1981
   Insitute University of Liverpool
3) First Name Ignacio
   Patronic Name
   Family Name de Gregorio
   Year Of Birth 1976
   Insitute University of Warwick
4) First Name Peter
   Patronic Name
   Family Name Giblin
   Year Of Birth 1943
   Insitute University of Liverpool
5) First Name Kevin
   Patronic Name
   Family Name Houston
   Year Of Birth 1968
   Insitute University of Leeds
6) First Name Daniel
   Patronic Name
   Family Name Littlestone

Year Of Birth 1982
Insitute University of Leeds
7) First Name David
Patronic Name
Family Name Mond
Year Of Birth 1950
Insitute University of Warwick
8) First Name Farid
Patronic Name
Family Name Tari
Year Of Birth 1963
Insitute University of Durham

2.6.5 Statistics

Number of Team Members involved in this project: 9
Number of Team Members under 35: 3
Number of Team Members who have individually received grants in INTAS projects: 0

2.7 Team : Indep. Univ. Moscow

2.7.1 Team Description

The team consists of leading specialists in the topological aspects of the singularity theory.
Two of them (Akhmetiev and Lando) are doctors of science, and the rest are candidates of Science.
Akhmetiev works intensively in realizing methods of algebraic topology in the terms of singularities of smooth maps.
Lando is well known for his works in topology of Hurwitz spaces. Duzhin and Lando have also many works on knots invariants via graph theory.
Merkov has important results in theory of generic plane curves and is, besides his mathematical scills, an excellent programmer.
Karpenkov has a series of works on the Morse theory (energy) approach to the study of the spaces of

2.7.2 List of publications

1 V.A.Vassiliev. Invariants and cohomology of first degree for spaces of embeddings of self-intersecting curves in R^n. Izvestiya RAS: Mathematics. 69:5 (2005), 3-52.
2 V.A.Vassiliev. Combinatorial calculation of combinatorial formulas for knot invariants. Transactions (Trudy) of Moscow Math. Society, 66(2005), 3-92 (to be translated by AMS).
3 M.E.Kazarian and S.K.Lando. On intersection theory on Hurwitz spaces, Izvestiya: Mathematics 69:5 (2004), 935-964
4 A.B.Merkov. Vassiliev invariants classify plane curves and doodles. Sbornik: Mathematics. 194:9 (2003), p. 31-62

5 P.M.Akhmetiev, D.Repovs and M.Cencel. On some algebraic properties of Cerf diagrams of one-parameter families of functions. Functional Analysis and its Applications 39:3 (2005), 1-13.

2.7.3 Team Leader and address

Title	Academician
Position	Senior Scientist
Sex	Male
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Patronic Name	Anatolievich
Family Name	Vassiliev
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Organisation / University / Institute	Independent University of Moscow
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Laboratory	
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2.7.4 List of Senior Scientists in the team

1) First Name	Petr
Patronic Name	Mikhailovich
Family Name	Akhmetiev
Year Of Birth	1963
Insitute	Institute of Terrestrial magnetizm, Ionosphere and Radiowave Propagation

2) First Name	Sergei
Patronic Name	Vasilievich
Family Name	Duzhin
Year Of Birth	1956
Insitute	St-Petersbourg branch of Steklov Mathematical Institute
3) First Name	Oleg
Patronic Name	Nikolaevich
Family Name	Karpenkov
Year Of Birth	1980
Insitute	Independent University of Moscow
4) First Name	Sergei
Patronic Name	Konstantinovich
Family Name	Lando
Year Of Birth	1955
Insitute	Independent University of Moscow
5) First Name	Alexandr
Patronic Name	Borisovich
Family Name	Merkov
Year Of Birth	1957
Insitute	Moscow Center of Continuous Mathematical Education

2.7.5 Statistics

Number of Team Members involved in this project: 6
Number of Team Members under 35: 1
Number of Team Members who have individually received grants in INTAS projects: 6

2.8 Team : Moscow State Univ.

2.8.1 Team Description

The team is based on the Dept. (Cathedra) of higher geometry and topology of the Faculty of Mechanics and Mathematics of the Moscow State University. It includes specialist in topology of singular spaces and maps, geometry of moduli spaces and mathematical Physics. The team includes 2 Dr.Sc., 3 PhD, 1 MA, and 1 graduate (PhD) student. The total list of the team is:
Prof. (Dr.Sc.) S.M.Gusein-Zade,
Dr.Sc. S.M.Natanzon,
Dr. (PhD) S.P.Chulkov,
Dr. (PhD) G.G.Ilyuta,
Dr. (PhD) V.N.Karpushkin,
Mr. (MA) B.Kazarnovski,
Mr. (MA, PhD student) K.P.Osminin.
The S.P.Chulkov and K.P.Osminin are under 35 years old.

2.8.2 List of publications

1 Poincare series of a rational surface singularity. A.Campillo, F.Delgado, S.M.Gusein-Zade. Inventiones mathematicae, 2004, v.155, no.1, 41-53.
2 Indices of vector fields or 1-forms and characteristic numbers. W.Ebeling, S.M.Gusein-Zade. Bulletin of the London Mathematical Society, 2005, v.37, no.5, 747-754.
3 Moduli of Riemann surfaces, real algebraic curves,
and their superanalogs. S.M.Natanzon. Translations of Mathematical Monographs, AMS, Vol.225 (2004), 160 pp.
4 Topological classification of Z^m_p actions on surfaces. A.Costa, S.M.Natanzon. Michigan Math.J., 50 (2002), N 3, 451-460.
5 A'Campo-Gusein-Zade diagrams as partially ordered sets. G.G.Ilyuta. Izvestiya: Mathematics, 2001, v.65, no.4, 49-66.

2.8.3 Team Leader and address

Title	Prof.
Position	Lecturer
Sex	Male
Date Of Birth	29/07/1950
First Name	Sabir
Patronic Name	Medzhidovich
Family Name	Gusein-Zade
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Organisation Registration Nr	
Academy / Branch	
Organisation / University / Institute	Moscow State University
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Laboratory	Cathedra of higher geometry and topology
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Website	

2.8.4 List of Senior Scientists in the team

1) First Name Sergey

Patronic Name	Pavlovich
Family Name	Chulkov
Year Of Birth	1980
Insitute	Moscow State University
2) First Name	Gennadiy
Patronic Name	Georgievich
Family Name	Ilyuta
Year Of Birth	1963
Insitute	Moscow State Open Pedagogical Institute
3) First Name	Vladimir
Patronic Name	Nikolaevich
Family Name	Karpushkin
Year Of Birth	1952
Insitute	Institute for Information Transmission Problems RAS
4) First Name	Boris
Patronic Name	Yakovlevich
Family Name	Kazarnovsky
Year Of Birth	1947
Insitute	Institute of SYstem Studies RAS
5) First Name	Sergey
Patronic Name	Mironovich
Family Name	Natanzon
Year Of Birth	1948
Insitute	Moscow State University
6) First Name	Konstantin
Patronic Name	Pavlovich
Family Name	Osminin
Year Of Birth	1983
Insitute	Moscow State University

2.8.5 Statistics

Number of Team Members involved in this project: 7
Number of Team Members under 35: 2
Number of Team Members who have individually received grants in INTAS projects: 5

2.9 Team : Steklov Inst. Math.

2.9.1 Team Description

The team consists of 4 experienced researchers and 1 young mathematician.
The team leader Vik.S.Kulikov is a well-known specialist in complex and real algebraic geometry, including geometry and topology of algebraic surfaces and their braid monodromy invariants.
A.G.Aleksandrov has his main works in theory of singularities and Pickard-Fuchs systems of differential equations.
M.M.Grinenko is a specialist in birational geometry and theory of Fano fibrations.
Val.S.Kulikov specialises in geometry of algebraic surfaces and theory of singularities.

The young member of the team, D.A.Stepanov (under 35), studies problems in toric singularities theory.
The structure of the team corresponds to the goals and the tasks of the project in its algebraic geometrical pa

2.9.2 List of publications

1 Vik.S.Kulikov and V.Kharlamov, On real structures of rigid surfaces, Izvestiya: Mathematics, 66:1, 2002
2 G.-M. Greuel and Vik.S. Kulikov, On symplectic coverings of the projective plane. Izvestiya: Mathematics, 69:4, 2005
3 Vik.S.Kulikov and Val.S.Kulikov, Generic coverings of the plane with $A-D-E$ singularities, Izvestiya: Mathematics, 64:6, 2000
4 A.G.Aleksandrov, Logarithmic differential forms, torsion differentials and residue. Complex Variables, v. 50 (2005), N 7–11, 777–802
5 M.M.Grinenko, Mori structures on a Fano threefold of index 2 and degree 1. Proc. Steklov Inst. Math. 2004, no. 3 (246), 103--128

2.9.3 Team Leader and address

Title	Prof.
Position	Senior Scientist
Sex	Male
Date Of Birth	13/04/1952
First Name	Viktor
Patronic Name	Stepanovich
Family Name	Kulikov
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Organisation Registration Nr	
Academy / Branch	Russian Academy of Sciences
Organisation / University / Institute	Steklov Mathematical Institute
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Website	

2.9.4 List of Senior Scientists in the team

1) First Name Aleksandr
   Patronic Name Grigorievich
   Family Name Aleksandrov
   Year Of Birth 1950
   Insitute Institute for Control Sciences
2) First Name Mikhail
   Patronic Name Mikhailovich
   Family Name Grinenko
   Year Of Birth 1968
   Insitute Steklov Mathematical Institute
3) First Name Valentin
   Patronic Name Stepanovich
   Family Name Kulikov
   Year Of Birth 1948
   Insitute Moscow State University of Printing
4) First Name Dmitry
   Patronic Name Anatolievich
   Family Name Stepanov
   Year Of Birth 1978
   Insitute Moscow State Technical University

2.9.5 Statistics

Number of Team Members involved in this project: 5
Number of Team Members under 35: 1
Number of Team Members who have individually received grants in INTAS projects: 4

2.10 Team : Moscow Aviation Inst

2.10.1 Team Description

Team leader - Prof. Vladimir Zakalyukin,
specialist in Singularities of Lagrangian and Legendre mappings.
Group, working in Global singularities:
-- Prof. M. Kazarian (Global Singularity theory, Thom polynomials, Gurwitz spaces),
-- Dr. V.Sedykh (Characteristic classes of wave fronts),
-- Dr. V.Chekanov (Legendre links, global Lagrangian singularities)
-- Dr. P.Pushkar (global Lagrangian and Legendre singularities).
Group working in applications of Lagrangian singularities:
- Prof. A.Davydov (singularities in control systems),
- Dr. I. Bogaevskiy (evolutions of wavefronts,
singularities in differential equations)
- Dr. O.Myasnichenko (singularities in subriemannian geometry)
Team paricipants are world leading specialists in these areas, already having tight contacts with teams from Netherlands, France, United Kingdom, Poland.

2.10.2 List of publications

1 1. Kazarian, M. E. Multisingularities, cobordisms, and enumerative geometry. (Russian) Uspekhi Mat. Nauk 58 (2003), no. 4(352), 29--88; translation in Russian Math. Surveys 58 (2003), no. 4, 665--724
2 2.A.A.Davydov, Generic Profit Singularities in Arnold s model of Cyclic Processes, Proceedings of the Steklov Institute of Mathematics, vol.250, 2005, pp. 79-94.

3 3. V.D.Sedykh, On the topology of singularities of the set of supporting hyperplanes of a smooth submanifold in an affine space, J. London Math. Soc. (2) {\bf 71} (2005), no.~1, 259--272.
4 4. Yu.V.Chekanov, P.E.Pushkar, Combinatorics of fronts of Legendrian links and the Arnold 4-conjectures, Russian Mathematical Surveys, Volume 60(2005), Number 1, Pages 95-149.
5 5. V.Goryunov, V.Zakalyukin, On stability of singular Lagrangian varieties, Functional Analysis and its Applications 38 (2004), n.4, 66-75.

2.10.3 Team Leader and address

Title	Prof.
Position	Group Leader
Sex	Male
Date Of Birth	09/07/1951
First Name	Vladimir
Patronic Name	Michailovich
Family Name	Zakalyukin
Organisation Type	Public
Organisation Registration Nr	
Academy / Branch	
Organisation / University / Institute	Moscow Aviation Institute
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Laboratory	
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2.10.4 List of Senior Scientists in the team

1) First Name Ilya
   Patronic Name Aleksandrovich
   Family Name Bogaevskiy
   Year Of Birth 1964
   Insitute Moscow Independent University
2) First Name Yuri
   Patronic Name Vitalievich
   Family Name Chekanov
   Year Of Birth 1964
   Insitute Moscow Independent University
3) First Name Aleksei
   Patronic Name Aleksandrovich
   Family Name Davydov
   Year Of Birth 1956
   Insitute Vladimirskii Gosudarstvennii Universitet
4) First Name Maxim
   Patronic Name Eduardovich
   Family Name Kazarian
   Year Of Birth 1965
   Insitute Steklov Mathematical Institute
5) First Name Oleg
   Patronic Name Mikhaylovich
   Family Name Myasnichenko
   Year Of Birth 1967
   Insitute Moscow Aviation Institute
6) First Name Petr
   Patronic Name Evgen'evich
   Family Name Pushkar'
   Year Of Birth 1972
   Insitute Moscow Independent University
7) First Name Vyasheslav
   Patronic Name Dmitrievich
   Family Name Sedykh
   Year Of Birth 1955
   Insitute Moscow Academy of Gaz and Oil

2.10.5 Statistics

Number of Team Members involved in this project: 8
Number of Team Members under 35: 1
Number of Team Members who have individually received grants in INTAS projects: 8

2.11 Team : Georgian Inst. Math.

2.11.1 Team Description

The following topics are currently developed by the members of team: effective methods of real algebraic geometry, specifically, algebraic formulae for computing topological and geometric invariants of algebraic sets and mappings, with applications to topology of configuration spaces of mechanical linkages and calculation of mean topological invariants of random real polynomial mappings; local and global topological aspects of nonisolated singularities, specifically, classification of singularities of codimension one and explicit formulae for counting singular points of various types in stable deformations of a given singularity; construction and classification of minimal round functions on manifolds; classification of singularities of holomorphic curves in loop spaces; topology of certain moduli spaces arising in complex analysis, specifically, singularities of configuration spaces of punctured Riemann surfaces and Yang-Mills connections; topology of moduli spaces of analytic discs attached to totally real submanifolds.

The first directon is developed by G.Khimshiashvili, T.Aliashvili, G.Giorgadze and G.Bibileishvili. The second one - by G.Khimshiashvili and M.Shubladze. The third - by G.Khimshiashvili and G.Giorgadze.

In particular, G.Khimshiashvili and T.Aliashvili investigate the topology of fibres of quadratic mappings. At present they work on estimates for the value of Euler characteristic of
the intersections of quadrics. They also obtained homotopy description of low-dimensional fibres of stable quadratic mappings in certain dimensions. They now plan to extend these results to higher dimensions and obtain effective formulae for the Betti numbers of fibres.

T.Aliashvili also works on estimation of the expected values of certain topological invariants of random polynomials and application of these estimates to invariants of random knots.

G.Bibileishvili works on estimating the topological degree of quasihomogeneous mappings.

G.Khimshiashvili also works on classification of minimal round functions of manifolds. In joint papers with D.Siersma, such functions were classified on surfaces and certain 3-folds. A fundamental problem is to describe the class of manifolds on which exist round functions. Recently, this topic appeared connected with singularities of holomorphic functions with values in loop spaces. A topical problem is to classify isolated singularities of holomorphic functions with values in loop spaces and there is good evidence that this problem can be attacked using methods of the theory of line singularities developed by D.Siersma.

G.Giorgadze investigates relations between the configuration spaces of planar polygons and moduli spaces of meromorphic connections in holomorphic bundles over Riemann surfaces.
Using this connection he computed the Euler characteristic of various moduli spaces and plans to extend these results to wider classes of moduli spaces.

2.11.2 List of publications

1 G.Khimshiashvili and D.Siersma, Remarks on minimal round functions, Banach Center Publ. 62, 2004, 159-172.

2 G.Khimshiashvili, Surfaces as intersections of quadrics, (Russian) Doklady Ross. Akad. Nauk 399, No.2, 2004, 173-175.
3 G.Khimshiashvili, Holomorphic tubes in Cauchy-Riemann manifolds, Complex Variables 50, No.7-11, 2005, 575-584.
4 T.Aliashvili and G.Khimshiashvili, Holomorphic dynamics in loop spaces, J. Dynam. Control Systems 12, No.1, 2006, 33-48.
5 G.Giorgadze, Quadratic mappings and configuration spaces, Banach Center Publ. 62, 2004, 73-86.

2.11.3 Team Leader and address

Title	Prof.
Position	Senior Scientist
Sex	Male
Date Of Birth	28/10/1951
First Name	Giorgi
Patronic Name	Nikolaevich
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2.11.4 List of Senior Scientists in the team

1) First Name Teimuraz

	Patronic Name	Murtazovich
	Family Name	Aliashvili
	Year Of Birth	1958
	Insitute	Georgian Technical University
2)	First Name	Gulnara
	Patronic Name	Georgievna
	Family Name	Bibileishvili
	Year Of Birth	1965
	Insitute	A.Razmadze Mathematical Institute
3)	First Name	Grigori
	Patronic Name	Karlovich
	Family Name	Giorgadze
	Year Of Birth	1960
	Insitute	Institute of Cybernetics, Georgian Academy of Sciences
4)	First Name	Mamuka
	Patronic Name	Shotaevich
	Family Name	Shubladze
	Year Of Birth	1957
	Insitute	Georgian Technical University

2.11.5 Statistics

Number of Team Members involved in this project: 5
Number of Team Members under 35: 0
Number of Team Members who have individually received grants in INTAS projects: 0

3 OBJECTIVES

3.1 RESEARCH OBJECTIVES

The aim of the Project is to investigate with the help of cooperation between different teams a wide range of problems in the Singularity Theory, in the Theory of Bifurcations, and in their applications. In particular, it is supposed (see more in research details):

- To construct and to study invariants of links and generic plane curves, to construct combinatorial formulae for such invariants. – - - To develop invariants separating double-periodic knots and detecting their reducibility with a special emphasis on the problem of detecting the knot invertibility by finite type invariants, to classify simplest periodic knots and links, to construct link invariants via graph theory.
- To study structures of Frobenius manifolds on spaces of deformations of complex algebraic curves, to construct and to investigate analogical structures, connected with real singularities.
- To study topological invariants of Hurwitz spaces (spaces of meromorphic functions), to investigate generating functions for these invariants and integrable systems corresponding to these generating functions.
- To study topological and analytical invariants of singular points of spaces, functions, ... (including Poincare series of filtrations on rings of functions and their connection with monodromy, integrations of motivic type and their applications, invariants of equivarian singularities, toric geometry).
- To study indices of singular points of 1-forms and of their collections in application to characteristic classes of singular varieties.
- To study structures on Riemann surfaces, to classify group actions on Riemann surfaces.
- To classify and to study singularities of shocks using Voronoi diagrams.
- To compute Thom polynomials in various global singularity theory problems.
- To investigate new invariants of Legendrian knots by Morse Theory and contact geometry.
- To study topology of stable corank 1 singularities of smooth mappings and its applications to geometrical problems.
- To investigate generic singularities of time averaged parametric optimization, in particular, the optimization of parametric cyclic processes.
- To study the geometry of caustics in sub-Riemannian nilpotent systems.
- To study simple singular Legendre varieties arising from classifcation of matrix families, and special generating families with applications to affine geometry and variational problems.
- To investigate various deformation problems arising in real algebraic geometry, in particular, real "dif=def" problem, description of braid monodromy factorizations of real pseudo-holomorphic curves, deformation of any complex singularity to a real one.
- To study particular questions related to the singularities of pairs theory, including the compactification problem for del Pezzo fibrations of small degree over a punctured disc, and description of dual complexes of singular points on three-dimensional algebraic varieties in the Mori category.
- To study logarithmic connections with poles along divisors and some of less known applications in the theory of singularities related with the theory of Gauss-Manin connection, multidimensional Picard-Fuchs systems and corresponding holonomic systems of differential equations.
- To develop formulae and algorithms for computing Betti numbers of the fibres of quadratic mappings, to work out procedures for computation the Betti numbers of real algebraic

varieties via reduction the defining equations to quadratic ones (with applications to estimates of the mean topological degree of random polynomials with a fixed Newton diagram, and computation of the Betti numbers of configuration spaces of planar polygons and graphs).
- To construct and classify minimal round functions on 3-folds, to obtain explicit formulae for the number of Morse points in real deformations of codimension one singularities, to compute Betti numbers of the Milnor fibre of codimension one singularities.
- To give an explicit description of a connection between critical loops of round functions and singularities of holomorphic functions with values in loop spaces of 3-folds.

3.2 Background and Justification

Singularity Theory, the study of bifurcations and monodromies are closely adjacent to a number of various problems

of calculus, algebraic geometry, topology,... In analysis they study structure of objects of general position and of objects of small codimension in functional spaces. In a number of cases corresponding problems of calculus originate from physical ones, say, mechanics and optics. Such problems often can be formulated in terms of singularities of dynamical systems and their bifurcations or in terms of singularities and bifurcations of Lagrangian and/or Legendre maps. In algebraic geometry they pay attention to singularities of algebraic (or analytic) varieties and of their maps. In topology Singularity Theory produces tools for study topological invariants of algebraic manifolds or varieties and of their maps and also methods to construct new invariants (as, say, in the knot theory). During last decades Singularity Theory benefited a lot from using methods of calculus, algebraic geometry, topology, and also influenced researches in these traditional fields of mathematics. As examples one can indicate new approaches to some traditional problems of algebraic geometry (e.g., the 16th Hilbert problem) and construction of new type of invariants in topology (e.g., invariants of finite order in knot theory). Singularity Theory, the study of bifurcations and monodromies traditionally developed within two-way influence and cooperation between the West European and fSU groups of researchers. An important role was played by the INTAS Projects: Investigations in singularity theory (94-4373, 1995-1997), Topology and analysis of discriminant sets (96-0713, 1997-1999), Singularity Theory and Bifurcations (00-0259, 2001-2003). All three Projects were coordinated by Prof. Dr. D.Siersma. In frames of those Projects there were elaborated a number of problems and there were obtained a number of new results (more than 300 papers were published or accepted for publication in framework of the Projects; many of them are joint papers of fSU and Western participants) and there was created a good scientific environment for further cooperation between West European and fSU groups of researchers. The groups involved in the Project include the majority of active researchers in the Singularity Theory from the fSU and from West and Central Europe. The Project is supposed to considerably extend the knowledge in the area by further developing existing problems and by studying a number of new ones in a cooperation between the groups.

According to the content of the research teams the supposed researches and directions of cooperation between West

European and fSU participants were divided into the following main themes (tasks):

Finite order invariants of maps of one-dimensional manifolds.

A huge family of invariants of links and generic plane curves, so-called finite type (or Vassiliev) invariants, arises from the singularity theoretical study of discriminant varieties in appropriate functional spaces. These invariants proved to be more powerful than all known polynomial link invariants; one of most intriguing problems is whether they can detect knot invertibility (while the polynomial invariants surely cannot). A new approach due to S.Duzhin has solved affirmatively the similar question concerning 2-string links (while previously it was solved for links with bigger number of components only) and promises also a progress in the problem concerning the knots.

First combinatorial formulas for knot invariants were introduced by J.Lannes and M.Polyak-O.Viro. V.Vassiliev has related the study of these formulas with the similar objects in the theory of plane arrangements and found an approach to algorithmic construction of such formulas; however this algorithm isn't programmed yet; also, the similar work for the parallel theory of generic plane curves waits to be accomplished.

Theory of periodic infinite links arises from the study of web types; it was initiated by experts from de Montfort University (UK) and St-Petersburg university of technology and design. A related problem concerns periodic links in the 3-space with (also periodic) surface removed. The problem of separating simplest such links can be, in principle, solved by suitably modified finite type invariants of knots in appropriate 3-manifolds. However, even for many standard webs this work is quite technically complicated. Also, the important problems of detecting irreducibility of a periodic link and of classification of simplest such links are waiting for solutions.

A basis for these studies was created, in particular, in the following publications:

S.K.Lando. On a Hopf algebra in graph theory. European J. of Combinatorics, Ser. B, Vol. 80 (2000), 104-121.

S.K.Lando. J-invariants of plane curves and framed chord diagrams. Funct. Anal. and its Appl., 40:1 (2006).
A.B.Merkov. Vassiliev invariants classify plane curves and doodles. Sbornik: Mathematics 194:9 (2003), 31-62.
V.A.Vassiliev. On invariants and homology of spaces of knots in arbitrary manifolds. In: AMS Transl. (2) Vol. 185 (1998), 155-182.
V.A.Vassiliev. Combinatorial calculation of combinatorial formulas for knot invariants. Trans. of Moscow Math. Society 66 (2005), 3-92.

Singularities and integrable systems.

Connectons between theory of singularity and integrable systems appear by two "mirror symmetry" canal. The first this is a description for spaces of versal deformations of singularities by solutions for some integrable systems. The second is a description of generating function for intersection indexes for strates of singularities by string solutions of some integrable systems. Both canals go to Frobenius manifolds. Two subtasks of this task correspond to these two approaches.

A nondegenerated differential geometry structure on the spaces of versal deformations of simple singularities was constructed by K.Saito in 80 years of last century. In 90 years B.Dubrovin found that these structures are some important examples of Frobenius manifolds, that independently arises in different partitions of mathematics and mathematical physics. A differential geometry structure, corresponding to the spaces of versal deformations of real singularity A_n, was constructed recently (S.Natanzon). This structure is a flat family of special noncommutative Frobenius algebras. Analytical description of noncommutative Frobenius manifolds leads to noncommutative integrable systems.

Recently Kazaryan, Lando and Zvonkine proved that generating functions for intersection numbers of the simplest geometrical homological classes of Hurwitz spaces satisfy integrable systems (KP, Toda). Relationship between Hurwitz numbers and intersection theory on moduli spaces (Ekedal, Lando, Shapiro, Vainshten) gives a possible to use these results for proof of Witten's conjecture about moduli spaces of r-spin curves.

A basis for these studies was created, in particular, in the following publications:

B.Dubrovin, Geometry of 2D topological field theories. In: Lecture Notes in Mathematics, v.1620, Springer-Verlag, Berlin, (1996), 120-348.
T.Ekedahl, S.K.Lando, M.Shapiro, A.Vainshtein. Hurwitz numbers and intersections on moduli spaces of curves. Invent. math., 146 (2001), 297-327.
Yu.Manin. Frobenius manifolds, Quantum cohomopogy and moduli spaces. v.47, American Math. Society, Colloquium Publication, 1999.
S.M.Natanzon. Extended cohomological field theories and noncommutative Frobenius manifolds. Geometry and Physics, 51:4, (2004), 387-403.
S.M.Natanzon. Moduli of Riemann surfaces, real algebraic curves, and their superanalogs. Translations of Mathematical Monographs, American Mathematical Society, Vol.225 (2004), 160 pp.

Topological and analytical invariants of singular spaces and maps.

The most attention will be paid to topological properties of singularities of analytic spaces and of their maps which play important role in problems of algebraic geometry and analysis, to their computation and relations. Relations between Poincare series and zeta functions of monodromies were found by S.Gusein-Zade, A.Campillo, F.Delgado, W.Ebeling. It was found that Poincare series of filtrations on rings of functions are in a natural way connected with the integration with respect to the Euler characteristic over the projectivisation of the ring of functions: a notion similar to and inspired by the notion of motivic integration invented by M.Kontsevich and studied by J.Denef and F.Loeser. The theory of index of a vector field led to the theory of indices of vector fields and 1-forms on singular varieties and to indices of collections of 1-forms corresponding to Chern numbers different from the Euler characteristic. The theory if indices has essential connections with the theory of characteristic classes of singular varieties. Geometry of Riemann surfaces and additional structures on them play important role in a variety of problems of mathematics and mathematical physics. Toric geometry plays essential role in studies of singularities; it is also connected with tropical geometry. Some topological invariants of singularities, such as M-morsifications of real singularitie, can be defined and/or expressed in terms of higher Bruhat orders (a generalization of the notion of permutation groups) and of higher partially ordered sets. Many properties of singularities and a number of applications of the Singularity Theory are connected with estimates of oscillatory integral.

A basis for these studies was created, in particular, in the following publications:

J.Denef, F.Loeser. Germs of arcs on singular algebraic varieties and motivic integration. Invent. Math. 135, no.1, 201-232.
A.Campillo, F.Delgado, S.M.Gusein-Zade. The Alexander polynomial of a plane curve singularity and integrals with respect to the Euler characteristic. International Journal of Mathematics, 2003, v.14, no.1, 47-54.
S.M.Gusein-Zade, I.Luengo, A.Melle-Hernandez. A power structure over the Grothendieck ring of varieties. Mathematical Research Letters, 2004, v.11, no.1, 49-58.
S.M.Gusein-Zade, W.Ebeling. Indices of 1-forms on an isolated complete intersection singularity. Moscow Mathematical Journal, 2003, v.3, no.2, 439-455.
A.Costa, S.Natanzon. Topological classification of Z^m_p actions on surfaces. Michigan Math. J., 50 (2002), no.3, 451-460.

Global invariants of singularities.

The program assumes to study many global topological properties of smooth mapping, in particular those which have general symplectic or contact nature. A crucial progress in this area was made by participants of the Project. In particular, M.Kazarian elaborated effective methods to extend the theory of Thom polynomials to various problems including classification of critical points of functions, Lagrange and Legendre singularities, multisingularities. Yu.Chekanov has constructed differential graded algebra which distinguishes non-equivalent Legendrian knots with equal classical invariants. Yu.Chekanov and P.Pushkar constructed combinatorial structures on the wavefronts of Legendrian knots. V.Sedykh found new conditions of a coexistence for stable singularities of corank 1. His main method is special resolution of wave front singularities. In fact, these results are the landmarks in recent developments of Global Singularity Theory.

A basis for these studies was created, in particular, in the following publications:
M.Kazarian. Classifying spaces of singularities and Thom polynomials. In: New developments in Singularity Theory (Cambridge 2000), NATO Sci.Ser. II Math.Phys.Chem, 21, Kluwer Acad. Publ., Dordrecht, 2001, 117-134.
M.Kazarian. Thom polynomials for Lagrange, Legendre and isolated hypersurface singularities. Proc. London Math. Soc. (3) 86 (2003) 707-734.
Yu.V.Chekanov. Invariants of Legendrian Knots. In: Proceedings of the International Congress of Mathematicians, vol.II, 385-394, Higher Education Press, 2002.
P.E.Pushkar. Maslov index and the symplectic Sturm theorems. Functional Anal. and Appl., v.32 (1998), no.3, 35-49
V.D.Sedykh. Resolution of corank 1 singularities of a generic front. Functional Anal. and Appl., v.37 (2003), no.2, 52-64.

Singularities of caustics and wavefronts related to geometric and applied problems.

During recent years the members of the team got a significant progress realising the V.I.Arnold general idea to simplify singularities which arise in various branches of mathematics and physics using symplectic and contact fibrations over the configuration space and studying the corresponding Lagrange of Legendre lifting of the singularity. The program covers study of topological properties of smooth mapping which have a general symplectic or contact nature and various applied problems involving special non-generic classes of Lagrangian and Legendrian submanifolds and varieties. Exactly these types of singularities are the most important for applications in differential equations, optimisation (in particular in mathematical economy, market analysis), mechanics, geometry (including computer vision anf robotics). These areas form the interface between the Singularity theory and all other multidisciplinary investigations involving singularities, shocks and catastrophes. Except global results mentioned above one should indicate the following ones. I.Bogaevsky investigated the low dimensional generic singularities of the matter evolution described by limit potential solutions with generic initial conditions. V.Zakalyukin created new methods to investigate singular Lagrange and Legendre projections applied to singularities of families of matrices, Minkowski symmetry set in affine geometry. New method based on flag contact singularities of A.Davydov and V.Zakalyukin provided classifications of singularities in multiparameter optimization. Results of A.Davydov on singularites of high order implicit differential equation and results of O.Myasnchenko on intergability of special subriemannian problems could be used to study caustics of exponential mappings in variational and control problems. The projects also assumes various applications of Lagrange of Legendre geometry being of the top interest.

A basis for these studies was created, in particular, in the following publications:

I.A.Bogaevsky. Perestroikas of shocks and singularities of minimum functions. Physica D: Nonlinear Phenomena, 2002, vol.173/1-2, 1-28.
A.Davydov, L.Ortiz-Bobadilia. Smooth normal forms of folded elementary singular points, J. Dynam. Control Syst. 1 (1995), 463-482.
O.Myasnichenko, Nilpotent (3,6) Sub-Riemannian Problem. Journal on Dynamical and Control Systems 8 (2002), no.4, 573-597.
P.J.Giblin, V.M.Zakalyukin. Singularities of families of chords. Funct. Anal. and Appl., 36 (2002), no.3, 63-68.
J.W.Bruce, V.V.Goryuynov, V.M.Zakalyukin, Sectional singularites and geometry of planar quadratic forms, Trends in mathematics: Trends in Singularity theory, Birkhauser, 2002, 83-97.

Deformation problems in real algebraic geometry.

"Dif=Def" problem in complex geometry asks: are two complex manifolds deformation equivalent if they are diffeomorphic as differentional manifolds. Recently M.Manetti constructed the first counterexamples to "Dif=Def" problem. Other counterexamples were constructed by V. Kharlamov and Vik. Kulikov using methods of real algebraic geometry. We (Vik.Kulikov in collaboration with V. Kharlamov) plan to investigate a real "Dif=Def" problem similar to one mentioned above and which is formulated as follows. Let (X_1,c_1) and (X_2,c_2) are two complex manifolds with real structures c_1 and c_2 (that is, c_1 and c_2 are anti-holomorphic involutions). Let X_1 and X_2 are deformation equivalent as complex manifolds and (X_1,c_1) and (X_2,c_2) are diffeomorphic as manifolds with involutions. Are they deformation equivalent as complex manifolds with anti-holomorphic involutions? This problem has affirmative solutions in the case of curves and for many special classes of surfaces (for rational surfaces (A. Degtyarev and V. Kharlamov), for real Abelian surfaces (essentially due to A. Comessatti), for geometrically ruled real surfaces (J.-Y. Welschinger), for real hyperelliptic surfaces (F. Catanese and P. Frediani), for real $K3$-surfaces (essentially due to V. Nikulin), and for real Enriques surfaces (A. Degtyarev and V. Kharlamov)). But, we think that in the general case this problem has negative solution.
In real algebraic geometry, there was the following problem: is any complex projective manifold deformation equivalent to one given over the field of real numbers? This problem has an affirmative answer for many types of projective manifolds. In 2002, Vik.S.Kulikov and V.Kharlamov constructed examples of projective surfaces which can not be deformed to real one. The questions about deformations of complex projective manifolds can be reduced to similar questions about deformations of plane caspidal curves, since if we consider a generic projection of a projective manifold of dimension greater than 1 to the projective space, then, by Chisini Conjecture proved by Vik.S.Kulikov, the manifold is uniquely determined by the branch locus of the projection. In the case of dimension 2, the branch locus of a generic projection is a plane caspidal curve whose type of imbedding to the projective plane (up to symplectic isotopy) is uniquely determined by its braid monodromy factorization type (BMFT). Therefore the problem: "If we know BMFT of a plane cuspidal curve, how to recognize in terms of BMFT can this curve be deformed to real one?" is very important for understanding what types of complex projective manifolds can not be deformed to real manifolds.
A basis for these studies was created, in particular, in the following publications:
V.Kharlamov and Vik.S.Kulikov. Diffeomorphisms, isotopies, and braid monodromy factorizations of plane cuspidal curves. C. R. Acad. Sci., Serie I, Paris, t.333, 5, 2001.
V.Kharlamov, Vik.S.Kulikov. Deformation inequivalent complex conjugated complex structures and applications, Turkish Jour. Of Math., Vol. 26, 2002.
V.Kharlamov, Vik.S.Kulikov, On Braid Monodromy Factorizations. Izvestiya: Mathematics, 67:3, 2003.
Vik.S.Kulikov. Old and new examples of surfaces of general type with $p_g=0$. Izvestiya: Mathematics, 68:5, 2004.
G.-M.Greuel, Vik.S. Kulikov. On symplectic coverings of the projective plane. Izvestiya: Mathematics, 69:4, 2005.

Singularities of pairs on algebraic threefolds.

Soon after the Mori theory have been introduced, in works of Ju.Kawamata, J.Kollar, M.Reid, V.Shokurov, etc. there appeared its logarithmic version, the so-called log minimal models program, Instead of direct consideration of terminal singularities as in the Mori theory, they involve log pairs (canonical divisor plus boundary) for two main reason: getting the common theory for singularities that more complicated than terminal, and applying the general results to birational geometry of singularities. This leads to the conseption of logartithmic pairs (log pairs). As the result, we have a unique basic theory for many methods and approaches that seem very different at the first look, e.g., the Sarkisov program and minimal blow-ups

of terminal points. Up to now, singularities of pairs remain one of the most actively developping subject in algebraic geometry. In the project we consider two problems related to singularities of pairs: compactification of del Pezzo fibrations and dual complexes of singular points.
For del Pezzo fibrations of small degree (1, 2, and 3) it is known (M.Grinenko, A.Pukhlikov) that non-singular compactifications are unique if they exist. On the other hand, there are examples of different compactifications if we allow cDv-points (i.e., gorenstein terminal singular points). In whole, the problem is not much studied. Only a few works (A.Corti for del Pezzo fibrations of degree not greater than 3, M.Manetti for fibrations into planes) are known on the subject.
The dual complex of a resolution generalizes the notion of the resolution graph of a surface singularity. It was first considered as a separate object by G. L. Gordon. It follows from the Weak Factorization Theorem of Abramovich-Karu-Matsuki-Wlodarczyk that the homotopy type of the dual complex does not depend on the choice of a resolution and can be considered as an invariant of the singularity. But this invariant is not described for high (>2) dimensional singularities yet, and it would be interesting to find it for different classes of singularities, in particular, for those appearing in the Minimal Model Program (terminal, canonical, log-terminal log-canonical). On the other hand, it follows from M. Artin s characterization of rational surface singularities that their resolution graphs are contractible in the topological sense, i.e, they have trivial homotopy type. It would be interesting to verify if this holds for rational singularities in higher dimension.
A basis for these studies was created, in particular, in he following publications:
M.M.Grinenko. On fibrations into del Pezzo surfaces. Math. Notes 69, No.4, 499-513 (2001).
M.M.Grinenko. On fibrewise modifications of fibrings into del Pezzo surfaces of degree 2. Russ. Math. Surv. 56, No.4, 753-754 (2001).
Corti, A. Del Pezzo surfaces over Dedekind schemes. Ann. Math. (2) 144, No.3, 641-683 (1996).
D.A.Stepanov. On the resolution of 3-dimensional terminal singularities. Math. Notes 77, No.1, 117-129 (2005).
D.A.Stepanov. Non-rational divisors over non-degenerate cDV-points. Sbornik: Math. 196, No.7, 1075-1088 (2005).
Logarithmic connections and multidimensional Picard-Fuchs systems.
The notion of logarithmic connection was defined by P. Deligne, he formulated and proved the theorem establishing a Riemann-Hilbert correspondence between monodromy groups and Fuchsian systems of integrable partial differential equations or flat connections on complex manifolds. He also gave a treatment of the theorem of Griffiths which states that the Gauss-Manin or Picard-Fuchs systems of differential equations are regular singular. The approach of Deligne was further developed by H. Esnault and E. Viehweg. Somewhat later, in 1990, C. Simpson constructed a moduli scheme for non-singular integrable connections on a projective variety and gave a number of applications to the nonabelian Hodge theory, to the theory of Higgs bundles, to the theory of Gauss-Manin connection, etc. However a moduli scheme for regular connections on a projective variety does not exist in general. This leads to the study of the moduli problem for logarithmic connections on a projective variety. Thus, in 1993, N. Nitsure generalized the method of Simpson and investigated this problem for semi-stable logarithmic connections. In 1976, K. Saito calculated the Gauss-Manin connection associated to the versal deformation of an A_3-singularity and described an elegant representation of the connection, he computed a set of integrable homogeneous meromorphic connections with logarithmic poles along the discriminant D of the minimal versal deformation of an A_3-singularity which contains the Gauss-Manin connection.
A basis for these studies was created, in particular, in he following publications:
A.G.Aleksandrov. Generalized Fuchsian systems of differential equations. Contemp. Math. and its applications, v.15 (Function theory), 2004, 3-13.
A.G.Aleksandrov. Logarithmic connections along a free divisor. In: Progress in Analysis, v. 2, World Sci. Publ., 2003, 705-716..
A.G.Aleksandrov. Moduli of logarithmic connections along a free divisor. In: Topology and Geometry, Contemporary Mathematics, v.314, 1-23, Providence, RI, AMS, 2003.
G.Giorgadze. On the Hamiltonians induced from a Fuchsian system. Mem. Differ. Equ. Math. Phys. 31, 69-82 (2004).
Topological invariants of real algebraic maps and varieties.
Both for theoretical and application purposes it is important to have effective methods for computing topological invariants of real algebraic varieties and mappings and semialgebraic sets. Solution to this problem is only known for the simplest topological invariants such as the mapping degree and Euler

characteristic which can be expressed as signatures of effectively constructible quadratic forms. A topical problem now is to develop effective methods of computing the number of components and Betti numbers of a given semialgebraic set. This problem is already nontrivial and significant for the fibres of real quadratic mappings and the research of the georgian team will be focused on developing effective methods for describing the topology of intersections of real quadrics. The final aim is to work out polynomial-time computer algorithms with respect to the number of variables. Recently, polynomial-time algorithms were suggested for computing the number of connected components of intersection of quadrics but no such algorithms are known for all Betti numbers. An important application of the methods and results of this research is to computation of the expected values of various topological invariants for random polynomial mappings. The expected value of the Euler characteristic of projectivized level surfaces and the mean gradient degree of rotation invariant Gaussian random homogeneous polynomial were found in the papers of I.Ibragimov, S.Podkorytov, G.Khimshiashvili. From the point of view of possible applications it is important to extend these results to other classes of random polynomials. In particular, it is planned to consider various classes of random polynomials with a fixed Newton diagram.

Another fundamental topic is the topological investigation of real nonisolated singularities. Classification and deformation theory were developed for isolated line singularities and singularities of codimension one in the complex case. Moreover, there were also obtained sufficiently detailed results on the structure of singularities of round functions which are real analogs of the isolated line singularities. However there is a lack of consistent deformation theory in the real case. As a first step in filling this gap it is planned to investigate stable deformations of real singularities of codimension one. An important problem in Hamiltonian mechanics and theory of integrable systems is to describe the topology of level surfaces and critical points of functions with smooth one-dimensional critical sets. Fundamental results on the connections between the critical sets of round Morse functions and topology of the underlying manifold were obtained by R.Bott, W.Thurston, and D.Asimov. However no such theory is known for non-transversally equisingular round functions. First steps in this direction were done in joint papers of G.Khimshiashvili and D.Siersma, where minimal round functions have been constructed on surfaces and 3-folds. A topical problem in this topic is classification of the minimal round functions on higher-dimensional manifolds. Another challenging problem is to investigate the recently discovered connection between round singularities and singularities of holomorphic functions with values in loop spaces.

A basis for these studies was created, in particular, in he following publications:

G.Khimshiashvili. Multidimensional residues and polynomial equations. Contemp. Math. Applic. 15, 2004, 71-120.

G.Khimshiashvili. New applications of algebraic formulae for topological invariants. Georgian Math. J. 1, No.4, 2004, 759-770.

G.Khimshiashvili. Surfaces as intersections of quadrics, Doklady: Mathematics 399, No.2, 2004, 173-175.

G.Khimshiashvili, D.Siersma. Remarks on minimal round functions, Banach Center Publ. 62, 2004, 57-68.

G.Khimshiashvili. Holomorphic tubes in Cauchy-Riemann manifolds, Complex Variables 50, No.7-11, 2005, 575-584.

T.Aliashvili, G.Khimshiashvili. Holomorphic dynamics in loop spaces, J. Dynam. Control Systems 12, No.1, 2006, 33-48.

M.Shub, S.Smale, Complexity of Bezout theorem. II: Volumes and Probabilities, Progress Math. 1993.

S.Basu, R.Pollack, M.-F.Roy. Algorithms in real algorithmic geometry, Springer-Verlag, 2003.

D.Grigoriev, D.Pasechnik. Polynomial-time computing over quadratic maps. I: Sampling in real algebraic sets. Computational Complexity 14, 2005, 20-52.

4 SCIENTIFIC / TECHNICAL DESCRIPTION

4.1 Research Programme

The aim of the project is a wide range of research in the Singularity Theory, study of bifurcations and monodromy. The programme of the Project includes the following tasks and subtasks: see more in Research details:

Finite order invariants of maps of one-dimensional manifolds.

Subtasks: Finite order invariants of knots determined by graph invariants. Detecting link orientation by finite type invariants. Periodic and double periodic knots and links. Combinatorial formulas for invariants and cohomology of generic submanifolds.

Singularities and integrable systems.

Subtasks: Noncommutative Frobenius manifolds. Hurwitz spaces and integrable systems.

Topological and analytical invariants of singular spaces and maps.

Subtasks: Poincare series, monodromy and integration of motivic type. Indices of 1-forms and characteristic classes of singular varieties. Structures on Riemann surfaces. Applications of higher Bruhat orders and multiplicative intersection theory.

Global invariants of singularities.

Subtasks: Calculation of Thom polynomials. Global Legendre topology. Coexistence of real singularities.

Singularities of caustics and wave fronts related to geometric and applied problems.

Subtasks: Applications of wave fronts and caustics. Singularities of wave transition. Singularities in optimization.

Deformation problems in real algebraic geometry.

Subtasks: "Dif=Def" problem in real algebraic geometry. Real braid monodromy factorisations. Equisingular deformation of a complex singularity to real one.

Singularities of pairs on algebraic threefolds.

Subtasks: Compactifications of del Pezzo fibrations of small degrees. Dual complex associated to a resolution of singularities.

Logarithmic connections and multidimensional Picard-Fuchs systems.

Subtasks: Logarithmic connection and uniformization equations for A_k-singularities. LC for special type of divisors and multi-dimensional Picard-Fuchs systems of differential equations

Topological invariants of real algebraic maps and varieties.

Subtasks: Algebraic formulae for topological invariants of real varieties. Topology of real nonisolated singularities.

4.2 Project Structure

4.2.1 Task Title : Finite order invariants of maps of one-dimensional manifolds

Task coordinator : Vassiliev, belonging to team: Indep. Univ. Moscow

Objectives :

Construction and study of invariants of links and generic plane curves, construction of combinatorial formulas for such invariants. Classification of double periodic infinite links and detection of their irreducibility. Detecting knot invertibility by finite type invariants. Construction of link invariants via graph theory.

Methodology :
Methods and tools include combinatorial and topological study of discriminant strata in spaces of curves, spectral sequences, theory of Hopf algebras, algebraic combinatorics, computer experiments

Task Input:
Theory of finite type knot invariants, including theory of weight systems and combinatorial formulas for these invariants; similar combinatorial formulas for (some) invariants of plane curves. Analogy btween knot theory and topological study of plane arrangements.

Result, milestones :
New explicit constructions of link invariants via graph theory; structure of related Hopf algebras. New constructions of weight systems. (Possibly) examples of finite type invariants detecting knot invertibility. Formal algorithms calculating combinatorial formulas for integer invariants of links and generic plane curves. Invariants of periodic infinite links, separating standard web types by these invariants, proof of irreducibility (or finding possible reductions) of these types.

4.2.1.1 Task Title : Finite order invariants of knots determined by graph invariants

Task coordinator : S.K.Lando, belonging to team: Indep. Univ. Moscow

Objectives :
Graph invariants are known to produce finite order knot invariants. The main objective of the subtask is the study of this fundamental relationship.

Methodology :
The methods of the study include
- search for graph invariants producing knot invariants;
- analysis of the algebraic structures on spaces of graph invariants producing knot invariants;
- computer experiments with graph invariants and their spaces;
- explicit computation of finite order knot invariants determined by Lie algebras;
- asymptotic estimation of dimensions of spaces of graph invariants.

Task Input:

The task is depending on : Finite order invariants of maps of one-dimensional manifolds
The relationship between graph invariants and knot invariants was first established by the present teams' members. Presently, both a complete (although not explicit) description of such invariants and important explicit examples of them are known. Computer software aimed for constructing and analysing graph invariants and algebraic structures on their spaces is developed.

Result, milestones :
The results can include
- explicit calculations of dimensions of the spaces of graph invariants under study;
- estimates for the asymptotic behavior of the dimensions;
- new examples of graph invariants producing knot invariants;
- explicit expression for graph invariants related to Lie algebras, in the simplest cases.

4.2.1.2 Task Title : Detecting link orientation by finite type invariants

Task coordinator : S.V.Duzhin, belonging to team: Indep. Univ. Moscow

Objectives :
We want to answer the question whether finite type invariants can detect the orientation of links with small number of components.

Methodology :
Our main method so far was to use the weight systems with values in the ad-invariant part of the universal enveloping algebras and their tensor powers. These spaces are studied by algebraic and combinatorial methods. In the case of links with more than one component, the ad-invariant subspace can be described in terms of circular necklaces whose orientation corresponds to the total orientation of a link.

Task Input:
Classical invariant polynomials, as well as quantum invariants in general, cannot detect the orientation of a knot. The class of Vassiliev invariants is strictly wider, and the problem whether it contains invariants that feel the knot orientation, is open for 15 years already. In the case of links, the corresponding problem was solved in the positive by X.-S.Lin (1996) for closed links with at least 6 components and by S.Duzhin and M.Karev for 2-component string links (2005).

Result, milestones :
We hope to answer the following questions:
- is there a finite type invariant that can detect the orientation of cloaed links with less than 6 components?
- if the answer to the previous question is positive, can it be applied to the orientation problem for knots?

4.2.1.3 Task Title : Periodic and double periodic knots and links

Task coordinator : Vassiliev, belonging to team: Indep. Univ. Moscow

Objectives :
Study and classification of periodic and double periodic tangled structures in the space, modelling the webs.

Methodology :
Extencions of finite type knot invariants and other known knot and link invariants to the case of infinite periodic structures in the space or in the space with some surface removed.

Task Input:
The importance of the study of periodic and double periodic tangled structures for the industry was recently noticed by very applied scientists working in de Montforf University (UK) and St-Petersbourg University of Design.

Result, milestones :
Construction of invariants distinguishing topologically different periodic and double periodic structures in the space. Proof of the non-equivalence of all classical structures occurring in the industrial web technology. Elaborating methods for proving irreducibility of such structures (or detecting the reducibility). Classification of such structures with small numbers of crosssing points inside the elementary domains. All the same in the theory of periodic tangled structures in the space with

(periodic) surface removed. Study of such structures by the methods of the Morse theory (of energy functional).

4.2.1.4 Task Title : Combinatorial formulas for invariants and cohomology of generic submanifolds

Task coordinator : Vassiliev, belonging to team: Indep. Univ. Moscow

Objectives :
Construction of combinatorial formulas for invariants and cohomology classes of spaces of knots, links or generic plane curves. Automatisation of this construction. Existence theorems of such formulas of different types.

Methodology :
Study of stratifications of discriminant varieties in spaces of curves in euclidean spaces. Simplicial resolutions of these varieties. Different methods of homology algebra.

Task Input:
Known combinatorial formulas, e.g. of Polyak-Viro type in the case of knots, and of arrow-segment (Merkov) diagrams in the case of plane curves. Analogous methods in the theory of plane arrangements.

Result, milestones :
Working algorithms for the construction of combinatorial formulas of these objects. Proof of the existence of combinatorial formulas of arrow-segment type for generic plane curves.
Possible generalizations to the cohomology spaces of other generic geometrical objects, including complex algebraic manifolds, aand embeddings of singular objects.

4.2.2 Task Title : Singularities and integrable systems

Task coordinator : S.M.Natanzon, belonging to team: Moscow State Univ.

Objectives :
Profound connectons between theory of singularity and integrable systems were found recently in mathematical physics research (String theory, Topological field theory and its deformations, mirror symmetry, etc). The project consists of investigations of these connectons. We plan also to use these connectons for construction new methods and results for theory of singularity and for theory of integrable systems.

Methodology :
We will be use algebraic methods (operads, A-infty algebras), differential geometry (including non-commutative), algebraic geometry and topology (moduli spaces and Hurwitz spaces), methods of
integrable systems (theta-function, string solutions). We will be work jointly with our Holland and French colleagues E.Looijenga, V.Turaev and D.Zvonkine.

Task Input:

The task is depending on : Structures on Riemann surfaces
Connectons between theory of singularity and integrable systems appear by two "mirror symmetry" canal. The first this is a description for spaces of versal deformations of singularities by solutions for some integrable systems. The second is a description of generating function for intersection indexes for strates of

singularities by string solutions of some integrable systems. Both canals go to Frobenius manifolds. Two subtasks of this task correspond to these two approaches.

Result, milestones :
We intend to develop a general theory of noncommutative Frobenius manifolds and to connect these structures with spaces of versal deformations of real singularity and with Gromov-Witten invariants of real algebraic manifolds. We intend also to find new explicit solutions for (possibly new) integrable hierarchies, describing the geometry of moduli spaces of meromorphic functions and its singularities.

4.2.2.1 Task Title : Noncommutative Frobenius manifolds

Task coordinator : S.Natanzon, belonging to team: Moscow State Univ.

Objectives :
Spaces of deformations of complex algebraic curves have natural structures of Frobenius manifolds. These structures reflect profound connections between singularities and mathematical physics. We intend to construct and investigate analogical structures connected with real singularities.

Methodology :
We use methods of noncommutative differential geometry, cohomological field theory and operads.

Task Input:

The task is depending on : Hurwitz spaces and integrable systems
A nondegenerated differential geometry structure on the spaces of versal deformations of simple singularities was constructed by K.Saito in 80 years of last century. In 90 years B.Dubrovin found that these structures are some important examples of Frobenius manifolds, that independently arise in different partitions of mathematics and mathematical physics: complex integrable systems, complex deformations of topological field theories, isomonodromic deformations of complex differential equations, complex Hurwitz spaces etc. An other source of Frobenius manifolds is Kontsevich-Manin cohomological field theory. This theory connects Frobenius manifolds with operads and topology invariants of complex manifolds: Gromov-Witten invariants, Mumford-Morita-Muller intersection numbers etc. The goal of our task is an elaboration of analogical theory for real manifolds. A differential geometry structure, corresponding to the spaces of versal deformations of real singularity A_n, was constructed recently (S.Natanzon). This structure is a flat family of special noncommutative Frobenius algebras. It arises also by a natural extension of a cohomological field theory on real algebraic curves (S.Natanzon). Analytical description of noncommutative Frobenius manifolds leads to noncommutative integrable systems.

Result, milestones :
We intend to develop a general theory of noncommutative Frobenius manifolds and to connect these structures with spaces of versal deformations of real singularity and with Gromov-Witten invariants of real algebraic manifolds.

4.2.2.2 Task Title : Hurwitz spaces and integrable systems

Task coordinator : S.Lando, belonging to team: Indep. Univ. Moscow

Objectives :

Spaces of meromorphic functions (Hurwitz spaces) are classical and actual object of investigation of mathematics and mathematical physics. A goal of the project is an investigation of topological invariants of Hurwitz spaces. Moreover, we hope to investigate generating functions for these invariants and integrable systems corresponding to these generating functions.

Methodology :
We use methods of global singularity theory (Thom-Kazaryan theory), theory of moduli spaces of curves, theory of integrable systems, theory of finite groups and their representations, computer experiments.

Task Input:

The task is depending on : Noncommutative Frobenius manifolds
Recently investigations (Kazaryan), developing of global singular theory of Thom, found important rules for manifolds of singularities. We hope to find some analogical results for multi singularities. We hope to use these results for development of intersection theory for geometrical homological classes of Hurwitz spaces. First reassuring results in this direction were recently found by Kazaryan, Lando and Zvonkine. These work proved that generating functions for intersection numbers of the simplest classes satisfy integrable systems (KP,Toda). We hope to find some analogical results for more large system of intersection numbers. Relationship between Hurwitz numbers and intersection theory on moduli spaces (Ekedal, Lando, Shapiro, Vainshten) gives a possible to use these results for proof of Witten's conjecture about moduli spaces of r-spin curves.

Result, milestones :
We intend to find new explicit solutions for (possibly new) integrable systems, describing
geometry for moduli spaces of meromorphic functions and its singularities.

4.2.3 Task Title : Topological and analytical invariants of singular spaces and maps

Task coordinator : S.M.Gusein-Zade, belonging to team: Moscow State Univ.

Objectives :
- To study Poincare series for some multi-index filtrations on rings of germs of functions and on the ring of polynomials and their relations to topological invariants of singularities.
- To study indices of singular points of 1-forms and of collections of 1-forms and their applications to the theory of characteristic classes of singular varieties.
- To elaborate topological classification of automorphisms groups and of r-spin structures of Riemann surfaces.
- To study combinatorial and topological properties of generalized Newton divided differences and interpolation series in connections with finite order functions.
- To construct multiplicative intersection theory for analytic germs.

Methodology :
Methods of algebraic geometry and algebraic topology (in particular, characteristic classes), integrals with respect to the Euler characteristic (in particular, over spaces of functions) and with respect to the generalized Euler characteristic (integrals of motivic type), Newton diagrams, toric geometry, group transformation methods, higher Bruhat orders, theory of Coxeter matroids.

Task Input:

The task is depending on : Topological invariants of real algebraic maps and

varieties
Connections between Poincare series of some filtrations on rings of germs functions on singular spaces and zeta functions of monodromy transformations. A method of computing Poincare series of some filtrations based on integration with respect to the Euler characteristic over projectivizations of spaces of functions. A geometric description of the λ-structure on the Grothendieck ring of complex varieties. Different definitions of indices of vector field, 1-forms, and collections of them. Theory of characteristic classes of singular varieties. Topological classification of some group actions on Riemann surfaces. Topological classification of r-spin structures on compact and non compact Riemann surfaces. Theory of generalized divided differences, generalized Manin-Schehtman discriminantal hyperplane arrangements. Toric multiplicative intersection theory.

Result, milestones :
- Poincare series of multi index filtrations on some rings of functions (in particular with group actions) and their motivic versions.
- Equivariant version of the λ-structure on the Grothendieck ring of varieties and its applications.
- Study of applications of the theory of indices of 1-forms and collections of them to characteristic classes of singular varieties.
- Topological classification of complex algebraic curves with abelian p-group of automorphisms and their real forms, topological classification of complex and real Gorenstein singularities.
- Multiplicative intersection theory for analytic germs.

4.2.3.1 Task Title : Poincare series, monodromy and integration of motivic type

Task coordinator : S.M.Gusein-Zade, belonging to team: Moscow State Univ.

Objectives :
- To compute Poincare series for some multi-index filtrations, in particular, on rings of germs of functions on spaces with group actions, on singularities defined by Newton diagrams, on the ring of polynomials of two variables (in the global setting),
- To study connections with topological invariants of singularities, including the zeta functions of monodromy transformations (in particular via the Saito duality). To study and to compute motivic versions of the Poincare series.
- To elaborate methods of computing zeta functions of monodromy transformations in global settings.
This has to be done mostly in cooperation of the team of the Moscow State University with the Spanish team (A.Campillo, F.Delgado, I.Luengo, A.Melle), the German team (W.Ebeling) and the Dutch team (D.Siersma).

Methodology :
Methods of algebraic geometry and topology, intgrals with respect to the Euler characteristic (in particular, over spaces of functions) and with respect to the generalized Euler characteristic (integrals of motivic type), Newton diagrams.

Task Input:
S.Gusein-Zade (team of the Moscow State University), A.Campillo and F.Delgado from the Spanish team and W.Ebeling from the German team found that there are (somewhat misterious) connections between Poincare series of some filtrations on rings of germs functions on singular spaces and topological invariants of singularities, such as zeta functions of monodromy transformations. There was elaborated a new method of computing Poincare series of some filtrations based on integration with respect to the Euler characteristic over projectivizations of spaces of functions (a notion similar to and inspired by the notion of motivic

integration). The goal is to find new situations and general settings when the discussed connections take place and possible explanations, to study motivic versions of these notions. There must be used a geometric description of the λ-structure on the Grothendieck ring of complex varieties given by S.Gusein-Zade, I.Luengo, A.Melle. The monodromy transformation in global settings (for polynomials, for meromorphic functions was studied in papers of D.Siersma (Dutch team), M.Tibar, S.Gusein-Zade, I.Luengo, A.Melle (Spanish team).

Result, milestones :
Computation of Poincare series of multi index filtrations on the rings of functions of some singularities defined by Newton diagrams, on spaces with group actions with applications to filtrations on factor spaces. Analysis of a possibility to use equivariant Poincare series for construction of an equivariant version of the monodromy zeta function. Generalized (?motivic?) Poincare series of some filtrations including those on the ring of polynomials of two variables. Equivariant version of the λ-structure on the Grothendieck ring of varieties and its application to generating series of invariants of orbifolds.

4.2.3.2 Task Title : Indices of 1-forms and characteristic classes of singular varieties

Task coordinator : S.M.Gusein-Zade, belonging to team: Moscow State Univ.

Objectives :
- To study indices of singular points of vector fields, of 1-forms and of collections of 1-forms on singular varieties. To find formulae for them.
- To study quadratic forms corresponding to germs of 1-forms on singular varieties generalizing the Eisenbud-Levine-Khimshiashvili one.
- To study possible applications of the theory of indices of 1-forms and of their collections to the theory of characteristic classes of singular varieties.

This has to be done mostly in cooperation of the team of the Moscow State University with the German (W.Ebeling) and the French (J.-P,Brasselet) teams.

Methodology :
Methods of algebraic geometry and algebraic topology (in particular, characteristic classes).

Task Input:

The task is depending on : Topological invariants of real algebraic maps and varieties
Definition of the index of a vector field on an isolated complete intersection singularity due to X.Gomez-Mond, J.Seade, A.Verjovski. A formula for it for a vector field on a hypersurface singularity. Extensions of this definition to singular points of 1-forms on isolated complete intersection singularities and an algebraic formula for this index, construction of indices of collections of 1-forms corresponding to Chern numbers different from the top one (in particular, of Chern obstructions), construction of quadratic forms corresponding to germs of 1-forms on an isolated complete intersection singularity (S.Gusein-Zade and W.Ebeling). Theory of characteristic classes of singular varieties (R.MacPherson, M.-H.Schwartz, J.-P.Brasselet and others).

Result, milestones :
Study of the quadratic forms corresponding to germs of 1-forms on isolated complete intersection singularities and their possible connections with indices of real 1-forms. Application of the theory of indices of 1-forms and collections of them to characteristic classes of singular varieties. Extension of the notions of the

indices of collections of 1-forms to meromorphic ones. Real versions of the notion of indices of collections of 1-forms and their applications.

4.2.3.3 Task Title : Structures on Riemann surfaces

Task coordinator : S.M.Natanzon, belonging to team: Moscow State Univ.

Objectives :

- Topological classification of automorphisms of group actions on Riemann surfaces.
- Classification and description of r-spin structures on Riemann surfaces with applications to Gorenstein singularities (Dolgachev) and gravitation theory (Witten).

We will be work jointly with our Spanish and Germain colleagues E.Bujalance, A.Costa, A.Porto de Silva, A.Pratoussevitch.

Methodology :

Topological methods, group transformation methods.

Task Input:

The task is depending on : Singularities and integrable systems

According to Hurwitz classical results, automorphisms group of any Riemann surface is finite. Moreover, any finite group is a automorphisms group of some Riemann surface. But it can act on different surfaces in topologically different ways. Topological classification of these actions is object of a lot of papers. This is a difficult problem which is not solved even for abelian groups. A full classification is known only for cyclic groups of order which is a degree of a prime number (Costa, Natanzon). In this case it is reduced to classification of symplectic structures on finite groups.

Topological classification of r-spin structures on compact and non compact Riemann surfaces was found recently by Jarvis, Natanzon and Pratoussevitch.

Result, milestones :

Topological classification of complex algebraic curves with abelian p-group of automorphisms and its real forms.

Topological classification of r-spin structures on real algebraic curves and topological classification of complex and real Gorenstein singularities, a description of their moduli spaces.

4.2.3.4 Task Title : Applications of higher Bruhat orders and multiplicative intersection theory

Task coordinator : G.G.Ilyuta, belonging to team: Moscow State Univ.

Objectives :

To study combinatorial and topological properties of generalized Newton divided differences and interpolation series in connections with finite order functions (analogues of Vassiliev invariants). To study the monodromy of real singularities by means of partially ordered sets theory, higher M-morsifications, higher Bruhat orders. To construct multiplicative intersection theory of analytic germs. To study divisibility of exponential sums connected with Mordell refinements of the ergodic theorem for toruses.

Methodology :

Theory of higher Bruhat orders, theory of Coxeter matroids, theory of generalized divided differences, toric geometry.

Task Input:
Description of topological invariants of singularities, such as M-morsifications of real singularitie, in terms of higher Bruhat orders (a generalization of the notion of permutation groups) and of higher partially ordered sets. Theory of generalized divided differences, generalized Manin-Schehtman discriminantal hyperplane arrangements. Toric multiplicative intersection theory which assigns to any pair of torus subvarieties of complementary dimensions not only the number of their common points but also the product of these points as torus elements.

Result, milestones :
To define and to investigate generalized higher Bruhat orders from the point of view of Coxeter matroids and Lascoux-Schutzenberger divided differences calculus. To describe connection of monodromy of real singularities with the theory of higher Bruhat orders and higher partially ordered sets. Multiplicative intersection theory for analytic germs as an analogy of the multiplicative intersection theory for torus subvarieties. Partial extension of such theory for reductive group algebraic subvarieties. Application of the multiplicative intersection theory to the Mordell-Lang conjecture.

4.2.4 Task Title : Global invariants of singularities

Task coordinator : M.E.Kazarian, belonging to team: Moscow Aviation Inst

Objectives :
Recently the team scientists achieved significant world leading progress in the global singularity theory which relates topological invariants of manifolds to the geometry of singularities of smooth maps. The objective of the further reseach dictated by the actual state-of–knowledge are:
-- to extend the theory of Thom polynomials to various classification problems in singularity theory,in particular for multisingularities, Hurwitz spaces;
-- to study the invariants of Legendre knots;
-- to consruct an equivariant version of the (Morse-Smale-Witten)-Floer theory;
-- to find a complet set of conditions of a coexistence for real stable singularities of corank 1.

Methodology :
The main method to evaluate Thom polynomials is due to M.Kazarian and it is based on the construction of the classifying space of multisingularities, which is related the R.Thom's construction for the classifying space of cobordisms.and can be applied to the study of the geometry of Hurwitz spaces.
The main new method to study Legendre knots is based on combinatorial study of generating family and on Chekanov's Theorem and Chekanov-type differential graded algebra for legendrian links.
The main method to study coexistence of corank 1 singularities is based on special resolution of singularities due to V.Sedykh.

Task Input:
The theory of characteristic classes dual to the loci of local singularities is known since 60's when R.Thom formulated the general principle about the existence of Thom polynomial. Kazarian extended the theory of Thom polynomials to the classification of Lagrange, Legendre, and isolated hypersurface singularities and multisingularities.
Yu.~Chekanov and Ya.~Eliashberg have constructed
differential graded algebra (DGA) which
distinguishes non-equivalent Legendrian knots with equal classical invariants.
Using the wave-front projections Yu.~Chekanov and

P.Pushkar constructed surprising combinatorial structures on the wavefronts of Legendrian knots. The four vertex Arnold's conjecture was proved.
A resolution of stable corank 1 singularities of wave front of A type was constractes and universal linear relations between the Euler characteristics of manifolds of corank 1 singularities of a generic wave front was found.

Result, milestones :
Extension of the theory of multisingularities to the cases of Lagrange, Legendre singularities, complete intersection singularities, proof of closed formulae with applications to enumerative geometry.
Elaboration of Morse theory for one and two parametrical families of functions, which is applicable for Legendre knots invariants and for construction of contact homologies.
Consruction of an equivariant version of the (Morse-Smale-Witten)-Floer theory suitable for the Floer functional on loops in a symplectic manifold.
Proof of the universal relations between Euler numbers of resolution manifolds of singularities A of corank 1.

4.2.4.1 Task Title : Calculation of Thom polynomials

Task coordinator : M.E.Kazarian, belonging to team: Moscow Aviation Inst

Objectives :
The global singularity theory relates topological invariants of manifolds to the geometry of singularities of smooth maps.
According to the R.Thom's principle the cohomology class Poincare dual to the a particular singularity locus can be expressed as universal polynomial in the Stiefel-Whitney classes of the manifolds. The objective is to extend the theory of Thom polynomials to various classification problems in singularity theory: classification of critical points of functions, isolated complete intersection singularities, Lagrange and Legendre singularities; real and complex singularities; local and multisingularities, Hurwitz spaces.

Methodology :
Although the theory of TP exists for about 50 years, its extension to the case of multisingularities have not been developed until recent work of M.Kazarian. The main method is based on the construction of the classifying space of multisingularities and to compute its cohomology group. The construction for the classifying space is related closely to the R.Thom's construction for the classifying space of cobordisms. This method has been also applied to the study of the geometry of Hurwitz spaces. As a result, a number of new relations between the cohomology classes of various strata has been established.

Task Input:
The theory of characteristic classes dual to the loci of local singularities is known since 60's when R.Thom formulated the general principle about the existence of Thom polynomial (TP). Many authors have contributed to the computation of the TP for particular classes of singularities. The classical approach used the resolution of singularities and application of the Gysin homomorphism. Later R.Rimanyi invented a simpler method based on the theorem about the existence of TP. The method of Rimanyi allowed to complete the computation of TP for almost all classified singularities. M.Kazarian extended the theory of Thom polynomials to the classification of Lagrange, Legendre, and isolated hypersurface singularities and multisingularities. This theory found many applications in the enumerative problems of algebraic projective geometry.

Result, milestones :

Extension of the theory of multisingularities to the cases of Lagrange, Legendre ingularities, isolated hypersurface singularities, complete intersection singularities, and other kinds of classification problems in singularity theory, proof of closed formulae.
Completion of the computation of residue polynomials of multisingularities for various classification problems.
Applications of the theory of multisingularities to the classical problems in enumerative geometry.
Elaboration of Global Singularity Theory approach to the study of other stratified spaces, e.g. moduli spaces of marked complex curves, and to the intersection theory on these spaces.

4.2.4.2 Task Title : Global Legendre topology

Task coordinator : Yu. Chekanov, belonging to team: Moscow Aviation Inst

Objectives :
The objectives are to study the invariants of Legendre knots, using the Lagrangian projection of a Legendre knot to symplectic plane, with applications to contact homologies for knots, and a new information of combinatorics of wave front projection of Legendrian knots. Also, a consruction of an equivariant version of the (Morse-Smale-Witten)-Floer theory suitable
for the Floer functional on loops in a symplectic manifold.

Methodology :
The main new method to be used is based on a combinatorial version of generating family for legendrian link in the standart contact space and to prove corresponding Chekanov-type theorem. It permits to study relations between augmentations of Chekanov-type differential graded algebra for legendrian links and generating function aproach to contact geometry. One of the main ingredients is a careful study of combinatorics for fronts in the cylinder.

Task Input:
Yu.Chekanov and Ya.Eliashberg have constructed differential graded algebra (DGA) which distinguishes non-equivalent Legendrian knots with equal classical invariants. In particular, they defined the corresponding contact homology groups and constructed examples of isotopic knots which are not-isotopic as Legendrian ones. Using the wave-front projection, Yu.Chekanov and P.Pushkar constructed surprising combinatorial structures on the wavefronts of Legendrian knots. There is an evidence that these tructures are closely related to the Volodin-Wagoner-Hatcher theory of pseudo-isotopies. The corresponding Legendrian K-theory hides certain homology groups of Legendrian knots similar to the Chekanov-Eliashberg contact homology. The work in this direction is in progress already. Theory of pseudo-involutions on wavefronts was constructed. It has numerous applications to contact and classical geometry -- the four vertex Arnold's conjecture and the four cusp conjecture was proved.

Result, milestones :
Completion of the generalisations of b Morse theory for a lower bound on the number of tangencies between a compact immersed submanifold of the Euclidean space and the hyperplanes defined by the levels of a height function. The recent studies in this direction suggest that there is a generalization when one deforms this one-parameter family of hyperplanes among one-parameter families of hypersurfaces (not necessarily the levels of a function), or more generaly among one-parameter families of wave fronts. The resulting theory is supposed to to absorb various estimates of Morse theory along with its relative analogs.
Elaboration of Morse theory for one and two parametrical families of functions

gives a possibility to construct corresponding K-theory, which is applicable for Legendre knots invariants and for construction of contact homologies.
Completion of the generalized theory of Maslov class.
Consruction of an equivariant version of the (Morse-Smale-Witten)-Floer theory suitable for the Floer functional on loops in a symplectic manifold. Subsequently, a construction of Floer type counterpart of the Ekeland-Hofer capacities. Proof of the product formula for them, which proves also a Hofer (1990) conjecture on product formula for the original Ekeland-Hofer capacities.

4.2.4.3 Task Title : Coexistence of real singularities

Task coordinator : V.D.Sedykh, belonging to team: Moscow Aviation Inst

Objectives :
The objectives are to investigate global invariants of coexistence of corank one singularites. The problem belongs to the global Singularity Theory. It is well-known that singularities of smooth mappings satisfy various conditions of a coexistence. There are a lot of results in this area. We are going to find new conditions of a coexistence for stable singularities of corank 1. The results will provide topological relations between manifolds of singular tangent hyperplanes of smooth submanifolds in the real projective n-space, between manifolds of supporting hyperplanes (hyperspheres) of smooth submanifolds in an affine n-space, between manifolds of singularities of the images of arbitrary smooth stable mappings of corank 1.

Methodology :
The main method is special resolution of singularities which was discovered recently in the partial case: for singularities of wave fronts. Recently a resolution of singularities of wave fronts of corank 1 (having only stable singularities of corank 1) was constructed.
It involves an iteration of the smooth proper coverings over the singularites of the initial map.

Task Input:
We have constructed a resolution of stable corank 1 singularities of wave front of A type.. Using this construction, we calculated universal linear relations between the Euler characteristics of manifolds of corank 1 singularities of a generic front.
The multiplicity of this covering does not depend on the dimension of the ambient space, on its topology, and also on the topology of the initial front.

Result, milestones :
Description of the properties of iterations of type A-transformations of wave fronts responsible for coexistance of singularities of the boundary of a hyperbolic connected component of the complement to a wave front of corank 1.
A generalization of the theory of wave fronts A type -transformations for arbitrary stable smooth mappings of corank 1. A description of properties of the respective resolution.
To elaborate applications of the above construction to the problem of a coexistence of singularities of the image of smooth mapping of corank 1.
Proof of the universal relations between Euler numbers of manifolds of these singularities. To elaborate applications to the problem of the coexistence of corank 1 multisingularities of smooth mappings.

4.2.5 Task Title : Singularities of caustics and wave fronts related to geometric and applied problems

Task coordinator : V.M.Zakalyukin , belonging to team: Moscow Aviation Inst

Objectives :

The present state of the art (meeting the actual demand of applied sciences) achieved by the team scientists, which are well known specialist in the area, implies the following objectives:

-- to describe new classes of stable Lagrangian and Legendre varieties arising in geometry and singularity theory;

-- delevop classifications of singularities of special wave fronts in geometry and control theory;

-- to investigate generic singularities of time averaged parametric optimization, in particular, the optimization of parametric cyclic processes;

--to classify generic bifurcations of local controllability of families of control systems on the surfaces;

-- to study Lagrangian singularities related to sub-Riemannian geometry;

-- to describe generic singularities of the matter evolution governed by limit otential solutions with generic initial conditions of Burgers equation.

Methodology :

The main methods are:

-- the theory of singularities of composed mappings, and algebras of wave front logarithmic vector fields;

-- the methods of D.Siersma theory for non-isolated singularities for Lagrangian singularities of exponential mappings in sub- Riemannian geometry;

-- the nilpotent integrable approximation methods in sub-Riemannian caustics;

-- averaged dynamics methods in cyclic processes optimisation;

-- Voronoi diagrams and the generalized viscosity solution methods.

Task Input:

The recent joint results with the members of the other teams show that the matrix singularities and some problems in geometry provide new examples of stable Lagrange varieties.

The sub-Rieamnnian singularities of caustic and wave fronts were studied for the case of growth vector (3,6) using SO(3) symmetry.

The classification of singularities and their transitions of limit potential solutions of the Burgers equation with generic initial conditions (in 2D and 3D space) was obtained based on special solutions of Hamilton-Jacobi equation.

The optimal strategies in average optimisation problems are proven to be either periodic or stationary, their lower dimensional generic singularites were classified.

Result, milestones :

Description of the logarithmic vector fields for the discriminant of stable Legendre variety.

Classification of simple ramified coverings over the union of a wave front and a suspension of bifurcation diagram of the function singularity.

Elaboration of the theory of Lagrangian generating families with non-isolated singularities.

The description of the caustic and the wave fronts (non-holonomic spheres) for general nilpotent(n, n(n+1)/2)sub-Riemannian problem.

Effective construction of the limit velocity fields of potential solutions of the Burgers equation with vanishing viscosity.

Classification of generic matter evolution singularities in three-dimensional space.

Classification of generic singularities of infinite horizon time averaged profit for one parameter cyclic process with the fixed period or in the presence of the discount rate.

Classification of generic bifurcations of local controllability of families of control systems on the surfaces.

4.2.5.1 Task Title : Applications of wave fronts and caustics

Task coordinator : V.M.Zakalyukin, belonging to team: Moscow Aviation Inst

Objectives :

The objectives are to describe new classes of stable Lagrangian and Legendre varieties arising from recently obtained classifications of simple singularities of families of symmetric and square matrices, classifications of generic singularities of families of chords in affine geometry.

These new objects are to be included into a consistent theory the basis of which was created in 90-th by A.Givental and V.Zakalyukin.

The theory will be accompanied by applications to classification of singularities of wave front metamorphoses in affine and projective geometries, translation invariant variation problems, control theory. Also the theory of multilocal generating families of Lagrangian and Legendre varieties of exponential mappings in sub-Riemannian geometry will be developed.

Methodology :

The main methods are the singularity theory of special equivalence on the space of deformations of hypersurfaces and complete intersection singularities, including the theory of singularities of composed mappings, and algebras of wave front logarithmic vector fields.

The methods of D.Siersma theory for non-isolated singularities are to be applied to the theory of Lagrangian singularities of exponential mappings in sub-Riemannian geometry.

Task Input:

The recent joint results with the members of the other teams shows that the simple symmetric matrix singularities are classified by special pairs of a Weil group A,D, E and its subgroup. The natural inducing mapping happens to be Legendre stable in the sense of A.Givental. (These new examples are important for investigations of D-modules and Frobenuous manifolds).

On the other hand, special Legendre varieties arising in affine geometry (in families of chords, in singularities of contact of a variety with its shifts by finite dimensional Lie group action) have also properties of Legendre stability.

Result, milestones :

Description of the logarithmic vector fields for the discriminant of the weighted homogeneous mapping inducing a stable Legendre variety from a weighed homogeneous versal deformation of a hypersurface singularity.

Classification of simple ramified coverings over the union of a wave front and a suspension of bifurcation diagram of the function singularity.

Classification of generic Legendre singularities of projective Minkowski symmetry set and its generalizations, classification of generic metamorphoses of degenerate wave fronts in variation problems with constraints.

Topological classification of systems of implicit ordinary differential equations based on Lagrange and Legendre stable varieties and flag contact singularities.

Completion of the of D.Siersma's type theory of non-isolated singularities of generating families of Lagrangian singularities of exponential mappings in sub-Riemannian geometry.

4.2.5.2 Task Title : Singularities of wave transition

Task coordinator : I.A.Bogaevski, belonging to team: Moscow Aviation Inst

Objectives :

The subject of investigation are viscosity vanishing limit potential solutions of Burgers equation. They are of interest in physics. For instance, they describe the formation of cellular structure of the matter in so-called adhesion model of matter evolution of the universe.. The objectives are:
- a description of generic singularities of the matter evolution described by limit potential solutions with generic initial conditions;
- an effective construction of the limit velocity field as a generalized derivative of the limit potential;
- a description of the limit motion of particles in inviscid solutions of the forced Burgers equation everywhere including the shock discontinuities.

Methodology :
According to our conjecture, the limit velocity at a singular point of the limit potential can be constructed with the help of the Voronoi diagram of some special points defined by the initial condition. The main new method is based on the generalized viscosity solutions not o the limit potential itself, but to the corresponding velocity field. This yields the possibility to classify generic singularities of the underlying matter evolution as well as their propagation.

Task Input:
Recently we got a classification of singularities and their transitions of limit potential solutions of the Burgers equation with generic initial conditions (in 2D and 3D space). This classification is based on an effective construction of a limit potential as a generalized (or viscosity) solution of the unforced Hamilton--Jacobi equation.

Result, milestones :
Effective construction of the limit velocity fields of potential solutions of the Burgers equation with vanishing viscosity.
Decription of the physically consistent model of the motion of particles in the classical variation problem with least Lagrangian action everywhere including the shocks and generalizations for arbitrary smooth convex Hamiltonians.
Classsification of generic singularities of the matter evolution in plane using the classification of Siersma of different types of the Voronoi diagrams of a few points.
Classification of generic matter evolution singularities in three-dimensional space.

4.2.5.3 Task Title : Singularities in optimization

Task coordinator : A.A.Davydov, belonging to team: Moscow Aviation Inst

Objectives :
The objectives are:
-- to investigate generic singularities of time averaged parametric optimization, in particular, the optimization of parametric cyclic processes. Up to now, the answeres are known only in some special cases (L.N.Bryzgalova, 1977, 1978; V.I.Matov, 1981; D.Siersma, V.I.Arnold, 2000, 2002; V.M.Zakaluykin and A.A.Davydov, 1997, 1999; A.A.Davydov, 2003) in spite of their importance for numerous applications (in chemistry, mechanics, biology);
-- to study Lagrangian singularities related to sub-Riemannian. Their caustics are the main objects of the study. They strongly differ from the Riemannian case (for example, the closure of a point exponential caustic contains the point itself) and are of significant interest.

Methodology :
The principle method in the study of sub-Riemannian caustics systems is to start with the study of their tangent spaces (after Gromov) which are Lie algebras with left invariant sub-Riemannian structures and then to consider the general case as

a deformation. This notion of the linearization developed by Gromov and Bellaiche provides a nilpotent Lie algebra provided with a left invariant sub-Riemannian structure. Basing on these inputs the following tasks (with applications in Control Theory and Non-holonomic Dynamics) will be performed.

Task Input:
The sub-Rieamnnian linearization was studied for the case of growth vector (3,6) using SO(3) symmetry. The singularities of caustic and the wave fronts (non-holonomic spheres) were determined. The SO(n)-symmetry of general nilpotent (n, n(n+1)/2) sub-Riemannian which implies the complete integrabilty of Hamiltonian system for Carnot - Caratheodory geodesics was used to state principle conjectures on the caustics structure.

Result, milestones :
The classifications of average parametric generic singularities for control systems or dynamic inequalities when both the parameter and the phase space are of low dimensions.
The investigation of generic singularities and their stability for local transitivity zones, the studying of maximum functions for generic families of functions over such zones with respect to phase variable.
The analysis of singularities of optimal cyclic motion and the transition between such a motion and the stationary optimal regime.
The description of the caustic and the wave fronts (non-holonomic spheres) for general nilpotent (n,n(n+1)/2)sub-Riemannian problem.
Classification of generic singularities of infinite horizon time averaged profit for one parameter cyclic process with the fixed period or in the presence of the discount rate.

4.2.6 Task Title : Deformation problems in real algebraic geometry

Task coordinator : Vik.S.Kulikov, belonging to team: Steklov Inst. Math.

Objectives :
We plan to investigate various deformation problems arising in real algebraic geometry, in particular, real "dif=def" problem, description of braid monodromy factorizations of real pseudo-holomorphic curves, deformation of any complex singularity to real one.

Methodology :
We use methods of real alegebraic geometry, the technique developed by Moisheson, Teicher, Kulikov, and Kharlamov on braid monodromy factorizations of plane curves, and the deformation theory of differential graded Lie algebras developed by M.Schlessinger, M.Koncevich, M.Manetti.

Task Input:
Last years we see an increasing interest to the deformation theory of compact real and complex manifolds, in particular, in connection to the “Dif=Def” problem. "Dif=Def" problem in complex geometry asks: are two complex manifolds deformation equivalent if they are diffeomorphic as differentional manifolds. Recently M.Manetti constructed the first counterexamples to "Dif=Def" problem. Other counterexamples were constructed by V. Kharlamov and Vik. Kulikov using methods of real algebraic geometry. The questions about deformations of complex projective manifolds can be reduced to similar questions about deformations of plane caspidal curves, since if we consider a generic projection of a projective manifold of dimension greater than 1 to the projective space, then, by Chisini Conjecture proved by Vik.S.Kulikov, the manifold is uniquely determined by the branch locus of the projection. In the case of dimension 2, the branch locus of a

generic projection is a plane caspidal curve whose type of imbedding to the projective plane (up to symplectic isotopy) is uniquely determined by its braid monodromy factorization type (BMFT). Therefore the problem: how to recognize in terms of BMFT can this curve be deformed to real one, is very important for understanding what types of complex projective manifolds can not be deformed to real manifolds.
What about the real “dif=def” problem, it has affirmative solutions in the case of curves and for many special classes.

Result, milestones :
Construction of examples of surfaces with negative answer for the real "dif=def" problem. Description of necessary conditions for braid monodromy factorization of plane algebraic curves to be a braid monodromy factorization of a curves defined over the field of real numbers. Development of theory of deformation of real singularities.
The results of the task are important for the problem of classification of real algebraic varieties.

4.2.6.1 Task Title : "Dif=Def" problem in real algebraic geometry

Task coordinator : Vik.S.Kulikov, belonging to team: Steklov Inst. Math.

Objectives :
Let X_1 and X_2 are deformation equivalent as complex manifolds and (X_1,c_1) and (X_2,c_2) are diffeomorphic as manifolds with involutions. Are they deformation equivalent as complex manifolds with anti-holomorphic involutions?

Methodology :
We use methods of real alegebraic geometry and the known descriptions of moduli spaces of certain classes of algebraic surfaces of general type.

Task Input:
"Dif=Def" problem in complex geometry asks: are two complex manifolds deformation equivalent if they are diffeomorphic as differentional manifolds. Recently M.Manetti constructed the first counterexamples to "Dif=Def" problem. Other counterexamples were constructed by V. Kharlamov and Vik. Kulikov using methods of real algebraic geometry. What about the real "dif=def" problem, it has affirmative solutions in the case of curves and for many special classes of surfaces (for rational surfaces (A. Degtyarev and V. Kharlamov), for real Abelian surfaces (essentially due to A. Comessatti), for geometrically ruled real surfaces (J.-Y. Welschinger), for real hyperelliptic surfaces (F. Catanese and P. Frediani), for real $K3$-surfaces (essentially due to V. Nikulin), and for real Enriques surfaces (A. Degtyarev and V. Kharlamov)).

Result, milestones :
We plan to construct examples of surfaces with negative answer for the real "dif=def" problem. The results of the subtask are important for the problem of classification of real algebraic surfaces.

4.2.6.2 Task Title : Real braid monodromy factorizations

Task coordinator : Vik.S.Kulikov, belonging to team: Steklov Inst. Math.

Objectives :
To find necessary conditions for braid monodromy factorization of plane algebraic

curves to be a braid monodromy factorization of a curves defined over the field of real numbers.

Methodology :
We use the technique developed by Moisheson, Teicher, Kulikov, and Kharlamov on braid monodromy factorizations of plane curves.

Task Input:
In real algebraic geometry, there was the problem: is any complex projective manifold deformation equivalent to one given over the field of real numbers? This problem has an affirmative answer for many types of projective manifolds. In 2002, Vik.S.Kulikov and V.Kharlamov constructed examples of projective surfaces which can not be deformed to real one.
The questions about deformations of complex projective manifolds can be reduced to similar questions about deformations of plane caspidal curves, since if we consider a generic projection of a projective manifold of dimension greater than 1 to the projective space, then, by Chisini Conjecture
proved by Vik.S.Kulikov, the manifold is uniquely determined by the branch locus of the projection. In the case of dimension 2, the branch locus of a generic projection is a plane caspidal curve whose type of imbedding to the projective plane (up to symplectic isotopy) is uniquely determined by its braid monodromy factorization type (BMFT). Therefore the problem: how to recognize in terms of BMFT can this curve be deformed to real one, is very important for understanding what types of complex projective manifolds can not be deformed to real manifolds.

Result, milestones :
We plan to find necessary conditions for braid monodromy factorization of plane algebraic curves to be a braid monodromy factorization of a curves defined over the field of real numbers. The results are imporatnt for the classification problem of real algebraic surfaces up to deformations.

4.2.6.3 Task Title : Equisingular deformation of a complex singularity to real one

Task coordinator : Val.S.Kulikov, belonging to team: Steklov Inst. Math.

Objectives :
We plan to investigate the following two problems:
- to apply the formalism of the extended deformation theory to real singularities, i.e., the approach to the deformation theory via differential graded Lie algebras;
- whether any complex singularity can be equisingular deformed to a real one.

Methodology :
We plan to use the deformation theory of differential graded Lie algebras developed by M.Schlessinger, M.Koncevich, M.Manetti and methods of real algebraic geometry.

Task Input:
Theory of deformation of singularities is one of the central parts of theory of singularities. It was developed by Kodaira, Spenser, Kuranishi, M.Artin, Schlessinger and others. The deformation theory became again a very active area of research. We have in mind the extended deformation theory, i.e. the approach to the deformation theory via differential graded Lie algebras developed by M.Schlessinger, M.Koncevich, M.Manetti. One of the interesting problems is to apply the formalism of the extended deformation theory to real singularities, that is, to the pairs (X,c), where (X,0) is a germ of an analytic space and c is an antiholomorphic involution acting on (X,0). Another interesting question to be investigated is the following: whether any complex singularity can be deformed to

a real singularity? Here under deformation we mean equisingular deformation, for example, in the sense of J.Wahl (the other approaches of the theory of equisingularity were developed by Zariski, Teisser and others). It is likely that the response to the question is in the affirmative for two-dimensional singularities. The reason for this is that a two-dimensional singularity is determined by an infinitesimal neighborhood of the exceptional curve of its resolution and that any curve can be deformed to a real curve. On the other hand, it is likely that the response to the question is in the negative in dimension >2.

Result, milestones :
Development of theory of deformation of real singularities. Varification of the conjecture that any two-dimensional singularity can be deformed to a real singularity. Construction of examples of complex singularities which can not be deformed to real ones.

4.2.7 Task Title : Singularities of pairs on algebraic threefolds

Task coordinator : M.Grinenko, belonging to team: Steklov Inst. Math.

Objectives :
Following the ideology of logarithmic pairs on algebraic varieties, we apply it to two objectives:
1. Compactification of del Pezzo fibrations of small degree;
2. Description of dual complex of singular points on algebraic varieties.
We consider them for three-dimensional varieties.

Methodology :
We have the common initial point for both problems: obtain the so-called log resolution and apply the log minimal models program. After that we apply various methods of algebraic geometry and topology to reach the solutions, as it is described in the corresponding subtasks.

Task Input:
Soon after the Mori theory have been introduced, it appears its logarithmic version, the so-called log minimal models program, in works of Ju.Kawamata, J.Kollar, M.Reid, V.Shokurov, etc. Instead of direct consideration of terminal singularities as in the Mori theory, they involve
log pairs (canonical divisor plus boundary) for two main reason: getting the common theory for singularities that more complicated than terminal, and applying the general results to birational geometry of singularities. This leads to the conseption of logartithmic pairs (log pairs). As the result, we have a unique basic theory for many methods and approaches that seem very different at the first look, e.g., the Sarkisov program and minimal blow-ups of terminal points. Up to now, singularities of pairs remain one of the most actively developping subject in algebraic geometry. The problems in the subtasks - compactification of del Pezzo fibrations and dual complexes of singular points - reflect the both sides of the logarithmic theory.

Result, milestones :
The results will naturally give an impact to one of the essential problem in mathematics: classification of the corresponding objects, in our case, three-dimensional del Pezzo fibrations and singularities in the log minimal model program. There are possible particular applications described in the subtasks.

4.2.7.1 Task Title : Compactifications of del Pezzo fibrations of small degrees

Task coordinator : M.Grinenko, belonging to team: Steklov Inst. Math.

Objectives :
Given a del Pezzo fibration of small (1 or 2) degree over a punctured disk, we find its different compactifications (not necessary non-singular) to a fibration over the whole disk. Usually we restricted by the condition for threefolds to be terminal, and for fibers to be embedded into the corresponding three-dimensional projective space (P(1,1,2,3) for degree 1, P(1,1,1,2) for degree 2). Also we are interested in birational map joining such compactifications.

Methodology :
In general, there are to much manners to compactify del Pezzo fibrations over a punctured disk. First, we show that any fibration can be compactified to a "good model": a threefold embeded into a direct production of a disk and a variety of fixed type (here our best candidats are P(1,1,2,3) for degree 1 and P(1,1,1,2) for degree 2 for this "fixed type"). Then we show that we can always suppose good models to have "minimal" (in some sense) singularities in the central fiber. It remains describe all these models and maps between them. We plan to use general methods of algebraic and birational geometry.

Task Input:
For del Pezzo fibrations of degree 1 and 2 it is known (M.Grinenko) that non-singular compactifications are unique if they exist. On the other hand, there are examples of different compactifications if we allow cDv-points (i.e., Gorenstein terminal singular points). In whole,
the problem is not much studied. Only a few works (A.Corti for del Pezzo fibrations of degree not greater than 3, M.Manetti for fibrations into planes, Yu.Prokhorov and P.Hacking for del Pezzo's of degree 5 and higher) is known on the subject.

Result, milestones :
Our best expectations are to find good models. The follwoing things become accessible for studying:
1. General problem of birational rigidity for del Pezzo fibrations of small degree.
2. Properties of anticanonical (and "shifted by fibers") algebras for these fibrations.
3. The classification problem for del Pezzo fibrations with possible look to properties of the fiber over the generic point (del Pezzo surfaces over a non-closed field).

4.2.7.2 Task Title : Dual complex associated to a resolution of singularities
Task coordinator : D.Stepanov, belonging to team: Steklov Inst. Math.

Objectives :
For any log-resolution of singularities we can associate the dual complex of the exceptional divisor. This object reflects the combinatory difficulty of singularities. We plan to describe the homotopy type of the dual complex for different types of singularities, in particular,
hypersurface singularities, rational singularities, singularities appearing in the Minimal Model Program (terminal, canonical, log-terminal etc.). Usually we start with the 3-dimensional case.

Methodology :
The fact that the homotopy type of the dual complex is well-defined is based on the Hironaka Resolution Theorem and Abramovich-Karu-Matsuki-Wlodarzcyk Weak Factorization Theorem in the Logarithmic Category. In the most cases we restrict to varieties over the field of complex numbers and apply analytic methods of studying the topology of singularities. If we take a resolution of a rational singularity, the condition of rationality can be reformulated in terms of some

cohomologies connected with the exceptional divisor. Then with the help of standard technique such as Hodge theory we can try to determine which restrictions on the homotopy type of the dual complex follow from the rationality. Also we can use classification results if they are known (as for 3-dimensional terminal singularities).

Task Input:
The dual complex of a resolution of singularities was first separately studied by G.L.Gordon with connection to monodromy in the families of algebraic varieties. However, the fact that the homotopy type of the dual complex is independent of the choice of a resolution was established only with a help of modern factorization theorem for birational maps (D. Stepanov). The dual complex can be studied for singularities defined over an arbitrary field of characteristic zero (where we know the Hironaka resolution), but we start with the case of complex numbers where plenty of results on topology of singularities (Milnor, Steenbrink and others) are known. We have already verified that the homotopy type of the dual complex of toric singularities is trivial.

Result, milestones :
We try to determine the homotopy type of the dual complex for as wide as possible class of singularities. At first we expect:
1. To find restrictions on the dual complex for rational singularities. In particular, we suppose that that the homotopy type of the dual complex for rational 3-dimensional hypersurface singularities is trivial.
2. To study the dual complex for resolutions of 3-dimensional terminal singularities.

4.2.8 Task Title : Logarithmic connections and multidimensional Picard-Fuchs systems

Task coordinator : A.Aleksandrov, belonging to team: Steklov Inst. Math.

Objectives :
To study logarithmic connections with poles along divisors and some of less known applications in the theory of singularities related with the theory of Gauss-Manin connection, multidimensional Picard-Fuchs systems and the corresponding holonomic systems of differential equations.

Methodology :
The investigation is based on ideas, methods and techniques of the modern cohomology theory of varieties, the theory of complex spaces, the deformation theory of singularities, the theory of logarithmic and multi-logarithmic differential forms, the theory of regular meromorphic forms, the theory of holonomic differential systems and theory of D-modules.

Task Input:
The notion of logarithmic connection appeared in a work by P. Deligne (1970) devoted to the study of the Riemann-Hilbert problem. He formulated and proved a theorem establishing a correspondence between monodromy groups and Fuchsian systems of partial differential equations or flat connections on complex manifolds. He also gives a treatment of the famous theorem of Griffiths which states that Gauss-Manin or Picard-Fuchs systems of differential equations are regular singular. The approach of Deligne was then developed by N. Katz (1970), H. Esnault and E. Viehweg (1986). Among other things they investigated also the classification problem of logarithmic connections. Somewhat later, C. Simpson (1990) constructed a moduli scheme for nonsingular integrable connections on a projective variety and considered applications to nonabelian Hodge theory, to the theory of Higgs and Yang-Mills bundles, to the theory of Gauss-Manin connexion,

etc. On the other hand, Gauss-Manin connexion and multidimensional Picard-Fuchs systems are naturally arose in relations with studies of deformations of singularities. Thus, from well-known results of K. Saito (1988) it follows that the Gauss-Manin connexion associated to the versal deformation of an isolated complete intersection singularity determines the logarithmic connection with poles along the discriminant of the deformation.

Result, milestones :
- description of the logarithmic connection, uniformization equations in the case of A_k-singularities and the corresponding holonomic systems of differential equations;
- description of logarithmic connections for special type of divisors, including quasihomogeneous Saito free divisors, and the corresponding multi-dimensional Picard-Fuchs systems of differential equations.

4.2.8.1 Task Title : Logarithmic connection and uniformization equations for A_k-singularities

Task coordinator : A.Aleksandrov, belonging to team: Steklov Inst. Math.

Objectives :
To describe the set of all integrable meromorphic connections with logarithmic poles along the discriminant of the versal deformation of an A_k-singularity. One can present any element of this set as a systems of differential equations of special kind called uniformization equations. Its solutions determine the primitive form of a singularity.

Methodology :
We use methods of deformation theory of singularities, theory of logarithmic differential forms, theory of holonomic differential systems and theory of D-modules.

Task Input:
The minimal versal deformation of an A_k-singularity.

Result, milestones :
The set of all integrable meromorphic connections with logarithmic poles along the discriminant the minimal versal deformation.

4.2.8.2 Task Title : LC for special type of divisors and multi-dimensional Picard-Fuchs systems of differential equations

Task coordinator : A.Aleksandrov, belonging to team: Steklov Inst. Math.

Objectives :
To describe the set of all integrable meromorphic connections with logarithmic poles along a quasihomogeneous Saito free divisor.

Methodology :
We use methods of theory of logarithmic differential forms, theory of holonomic differential systems and theory of D-modules.

Task Input:
A quasihomogeneous Saito free divisor.

Result, milestones :

The set of all integrable meromorphic connections with logarithmic poles along the divisor.

4.2.9 Task Title : Topological invariants of real algebraic maps and varieties

Task coordinator : G.Khimshiashvili, belonging to team: Georgian Inst. Math.

Objectives :
To develop effective methods of computing basic topological invariants of real algebraic varieties given by explicit equations. At the first stage it is planned to work out effective algorithms for computing the Betti numbers of fibres of real quadratic mappings. Special attention will be given to the fibres with nonisolated singular sets. It is also planned to estimate and compute the mean values of various topological invariants of random polynomials and random polygonal knots.

Methodology :
The theoretical basis for effective computation of topological invariants of real varieties is given by the algebraic formulae for the Euler characteristic and mapping degree developed by G.Khimshiashvili, D.Eisenbud with H.Levine, Z.Szafraniec, J.Bruce, N.Dutertre. The Betti numbers of the fibres of quadratic mappings can be computed using results of A.Agrachev and R.Gamkrelidze, G.Khimshiashvili, D.Grigoriev and D.Pasechnik. The topological invariants of real varieties can be related to those of their singular sets using results of V.Vassiliev, M.Kazarian, S.Gusein-Zade and W.Ebeling, A.Aleksandrov, G.Khimshiashvili and D.Siersma.

Task Input:

The task is depending on : Topological and analytical invariants of singular spaces and maps
The basic tools for the planned research are provided by: signature formulae for topological invariants developed by G.Khimshiashvili, and D.Eisenbud with H.Levine; fast computer algorithms for calculating the topological invariants of quadratic mappings developed by D.Grigoriev and D.Pasechnik; deformation theory and surgery for round functions developed by G.Khimshiashvili and D.Siersma; general method of computing the mean values of rotation invariant Gaussian random polynomials suggested by M.Shub and S.Smale.

Result, milestones :
Algebraic methods and fast computer algorithms for calculating topological invariants of real varieties will be developed. These techniques will be applied to computing the topological invariants of configuration spaces of planar linkages and weighted graphs. Critical fibers of Seifert fibrations will be represented as singularities of holomorphic functions with values in loop spaces of three-dimensional manifolds. The expected values of basic topological invariants of rotation invariant Gaussian random polynomials will be computed and applications to estimating the average self-linking number and average crossing number of random polygonal knots will be worked out.

4.2.9.1 Task Title : Algebraic formulae for topological invariants of real varieties

Task coordinator : G.Khimshiashvili, belonging to team: Georgian Inst. Math.

Objectives :
To develope effective methods of computing topological invariants of the fibres of real algebraic mappings. As a first step, it is planned to develop algorithms for computing the Betti numbers of fibres of proper quadratic mappings in terms of

numerical invariants of the corresponding pencil of real quadrics. A closely related problem is to obtain exact estimates for the range of Euler characteristic and Betti numbers of intersections of quadrics for a fixed number of variables and quadrics. Special attention will be given to the topology of fibres of stable quadratic mappings and intersections of three real quadrics. It is also planned to estimate the computational complexity of arising algorithms. The ultimate goal is to obtain polynomial-time algorithms for computing the Betti numbers in the quadratic case.

Methodology :
The main tools for the planned investigation are given by the signature formulae for topological invariants (G.Khimshiashvili, D.Eisenbud, H.Levine, Z.Szafraniec, J.Bruce) and spectral sequences for computing homology of the fibres of real quadratic mappings (A.Agrachev). Estimates for the Euler characteristic can be obtained using various generalizations of Petrovsky-Oleynik inequalities obtained by V.Arnold, A.Khovanski, V.Kharlamov, and G.Khimshiashvili. Fast algorithms for computing Betti numbers in the quadratic case can be worked out using signature formulae and polynomial-time algorithms for counting the components of intersection of quadrics developed by D.Grigoriev and D.Pasechnik.

Task Input:

The task is depending on : Indices of 1-forms and characteristic classes of singular varieties
Among the prerequisites and preliminary results most closely related to this task one could mention the algebraic formulae for mapping degree and Euler characteristic developed by G.Khimshiashvili, D.Eisenbud with H.Levine, Z.Szafraniec, and J.Bruce; spectral sequences for computing homology of the fibres of quadratic mappings suggested by A.Agrachev; estimates for the Euler characteristic of intersections of quadrics obtained by T.Aliashvili and G.Khimshiashvili.

Result, milestones :
Effective algorithms for computing the Betti numbers of fibres of real polynomial mappings. Algebraic formulae for counting points in an explicitly given finite semi-algebraic subset. Exact estimates for the Euler characteristic of intersections of quadrics with a fixed number of variables and quadrics. Topological classification of compact intersections of three real quadrics. Polynomial time algorithms for computing topological invariants of the fibres of real quadratic mappings.

4.2.9.2 Task Title : Topology of real nonisolated singularities

Task coordinator : G.Khimshiashvili, belonging to team: Georgian Inst. Math.

Objectives :
To develop classification and deformation theory of real nonisolated singularities of various types. A conceptual problem is to obtain explicit formulae for the number of isolated singularities of various types appearing in stable deformations of a given real nonisolated singularity. A topical concrete problem is to develop deformation theory for real singularities of codimension one and describe the topology of their Milnor fibres. Another topical problem is to complete classification of minimal round functions on three-dimensional manifolds started by G.Khimshiashvili and D.Siersma. A closely related problem is to classify and give geometric models for critical points of holomorphic functions with values in loop spaces of three-dimensional manifolds.

Methodology :
The basic methods and tools for the planned investigation are given by general

theory of nonisolated singularities developed by D.Siersma and his collaborators. Especially relevant for fulfilling the foregoing tasks are deformation theory for isolated line singularities developed by D.Siersma and results on the topological structure of codimension one singularities obtained by M.Shubladze. Another important ingredient is given by the geometric description of holomorphic curves in loop spaces and contact boundaries of isolated plane curve singularities developed by J.-L.Brylinski, L.Lempert, M.Tibar, and G.Khimshiashvili.

Task Input:

The task is depending on : Topological and analytical invariants of singular spaces and maps
Deformation theory for nonisolated singularities developed by D.Siersma and R.Pellikaan. Surgery for minimal round functions developed by G.Khimshiashvili and D.Siersma. Geometric description of analytic discs in loop spaces obtained by T.Aliashvili and G.Khimshiashvili.

Result, milestones :
Characterization of manifolds admitting round functions. Construction of minimal round functions and classification of their typical singularities. Algebraic formulae for the number of Morse points appearing in a stable deformation of a given real nonisolated singularity. Realization of singular fibres of Seifert fibrations as singularities of holomorphic curves in loop spaces of 3-folds.

4.3 Project Management

4.3.1 Planning & Task allocation

4.3.1.1 List of Task Titles

1. Finite order invariants of maps of one-dimensional manifolds
   - 1.1 Finite order invariants of knots determined by graph invariants
   - 1.2 Detecting link orientation by finite type invariants
   - 1.3 Periodic and double periodic knots and links
   - 1.4 Combinatorial formulas for invariants and cohomology of generic submanifolds
2. Singularities and integrable systems
   - 2.1 Noncommutative Frobenius manifolds
   - 2.2 Hurwitz spaces and integrable systems
3. Topological and analytical invariants of singular spaces and maps
   - 3.1 Poincare series, monodromy and integration of motivic type
   - 3.2 Indices of 1-forms and characteristic classes of singular varieties
   - 3.3 Structures on Riemann surfaces
   - 3.4 Applications of higher Bruhat orders and multiplicative intersection theory
4. Global invariants of singularities
   - 4.1 Calculation of Thom polynomials
   - 4.2 Global Legendre topology
   - 4.3 Coexistence of real singularities
5. Singularities of caustics and wave fronts related to geometric and applied problems

5.1 Applications of wave fronts and caustics
5.2 Singularities of wave transition
5.3 Singularities in optimization

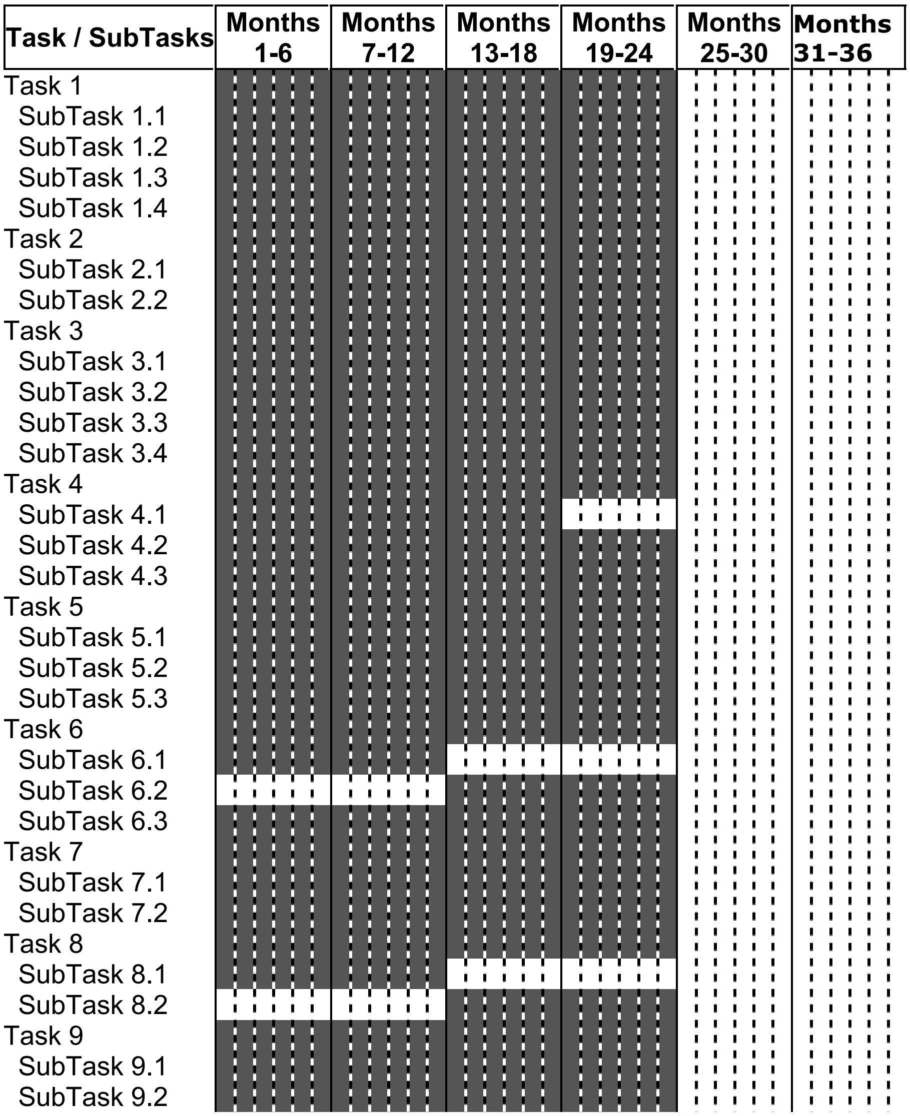
6. Deformation problems in real algebraic geometry
    6.1 "Dif=Def" problem in real algebraic geometry
    6.2 Real braid monodromy factorizations
    6.3 Equisingular deformation of a complex singularity to real one

7. Singularities of pairs on algebraic threefolds
    7.1 Compactifications of del Pezzo fibrations of small degrees
    7.2 Dual complex associated to a resolution of singularities

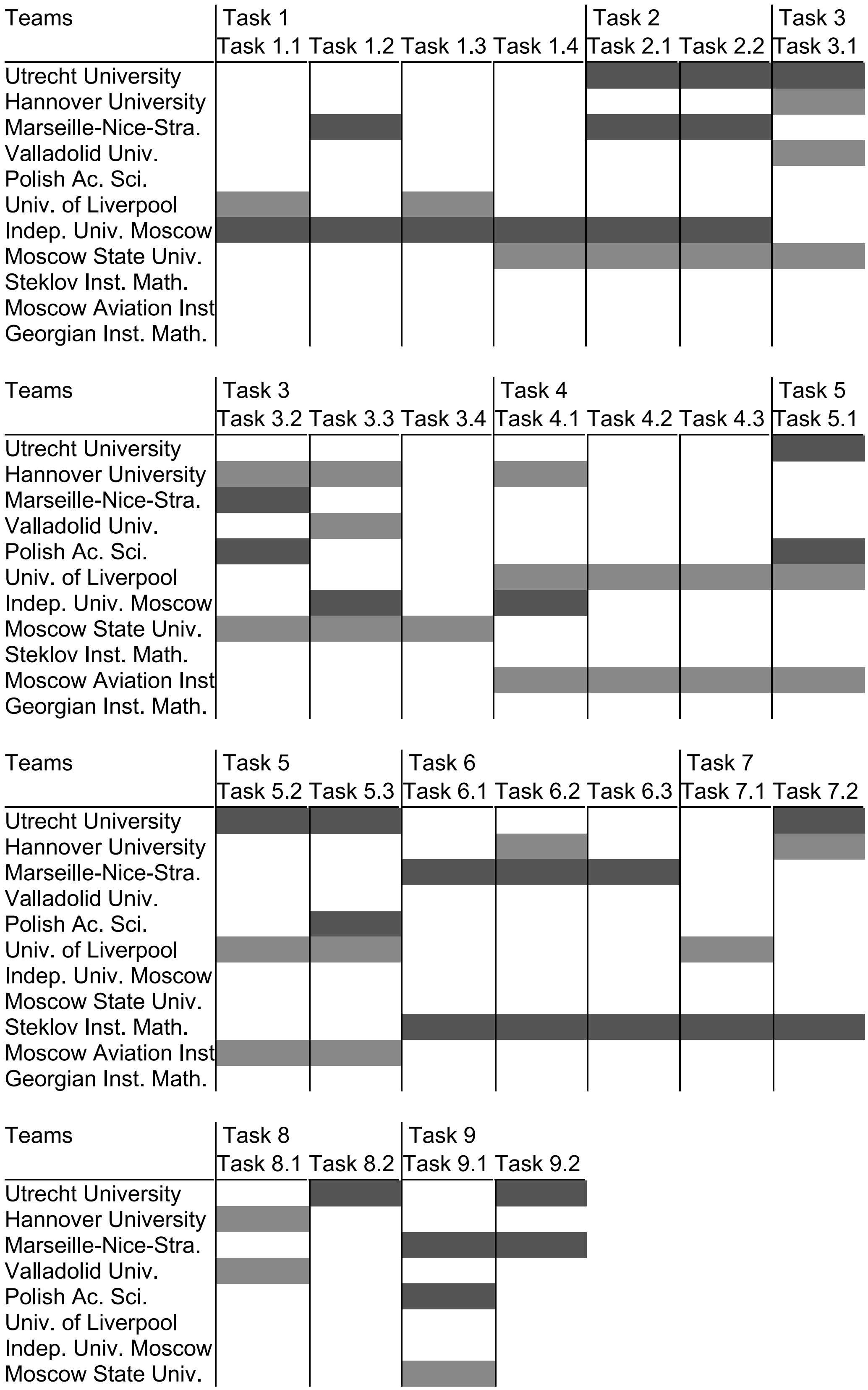
8. Logarithmic connections and multidimensional Picard-Fuchs systems
    8.1 Logarithmic connection and uniformization equations for A_k-singularities
    8.2 LC for special type of divisors and multi-dimensional Picard-Fuchs systems of differential equations

9. Topological invariants of real algebraic maps and varieties
    9.1 Algebraic formulae for topological invariants of real varieties
    9.2 Topology of real nonisolated singularities

4.3.1.2 The project will last 24 months with the activities as indicated in the diagram below



4.3.1.3 Team involvement



4.3.2 Project Management Description

The central management will be under supervision of the Coordinator Prof. Dr. Dirk Siersma done by the Mathematical Department of the Utrecht University. This Department has an experienced staff in managing international cooperations and good relations with the INTAS-management. The Coordinator and Prof. S.Gusein-Zade, who represents the fSU partners, will togeteher carry out the tasks of monitoring and control of the project, both scientific and financial. There will be regular contacts between them (email and in person). The system of management and cooperation was somewhat elaborated during previous INTAS Projects in which most part of the team members have participated.

Regular discussions of obtained results will take place at the weekly seminar at the Moscow State University. Cooperation between fSU and Western participants and joint researches will take place during visits of the participants to the INTAS countries, to Russia and Georgia and also by exchanging scientific information.

At least one meeting between key researchers from the teams is foreseen each year. A more wide meeting of the Project participants is planned for summer 2007 in the framework of the Conference on Singularity Theory and adjacent subjects devoted to the 70th birthday of V.I.Arnold to be held in Moscow.

Special attention will be for the exchange of for young scientists in order to give them the possibility to work at the other nodes. We intend to increase this by using other sources (e.g. bilatural agreements, such as the existing project between Netherlands and Russia).

The Coordinator will open a seperate website for exchange of information, e.g. about results, preprints, publications, meetings and a discussion board .

4.4 Project costs

4.4.1 Cost Table

The breakdown of costs of the INTAS contribution (in EURO) is given in the tables below.

INTAS MEMBER STATE TEAMS								
	Team name	Cost categories						TOTAL
		Labour Costs	Overheads	Travel & subs.	Consumables	Equipment	Other	(EURO)
1	Utrecht Unive...	0	2000	5500	0	0	0	7500
2	Hannover Univ...	0	0	6000	0	0	0	6000
3	Marseille-Nic...	0	0	6000	0	0	0	6000
4	Valladolid Un...	0	0	6000	0	0	0	6000
5	Polish Ac. Sc...	0	0	6000	0	0	0	6000
6	Univ. of Live...	0	0	6000	0	0	0	6000
SUBTOTAL	(EURO)	0	2000	35500	0	0	0	37500

NIS TEAMS								
	Team name	Cost categories						TOTAL
		Labour Costs	Overheads	Travel & subs.	Consumables	Equipment	Other	(EURO)
7	Indep. Univ. ...	15750	0	7130	0	0	0	22880
8	Moscow State ...	16650	0	8140	0	0	0	24790
9	Steklov Inst....	13200	0	5870	0	0	0	19070
10	Moscow Aviati...	20850	0	9660	0	0	0	30510
11	Georgian Inst...	10050	0	5200	0	0	0	15250
SUBTOTAL	(EURO)	76500	0	36000	0	0	0	112500

TOTAL	(EURO)	76500	2000	71500	0	0	0	150000

4.4.2 Justification of Costs
4.4.2.1 Labour costs (only for NIS teams)

Team name: Indep. Univ. Moscow

Number of individual grants		Cost per month	Total number of man months	Total cost (EURO)
Team Leader	1	200	15	3000
Senior Researcher	5	170	75	12750
Scientist/Engineer	0	0	0	0
Technical or Other	0	0	0	0
TOTAL				15750

Team name: Moscow State Univ.

Number of individual grants		Cost per month	Total number of man months	Total cost (EURO)
Team Leader	1	200	15	3000
Senior Researcher	4	170	60	10200
Scientist/Engineer	2	150	23	3450
Technical or Other	0	0	0	0
TOTAL				16650

Team name: Steklov Inst. Math.

Number of individual grants		Cost per month	Total number of man months	Total cost (EURO)
Team Leader	1	200	15	3000
Senior Researcher	4	170	60	10200
Scientist/Engineer	0	0	0	0
Technical or Other	0	0	0	0
TOTAL				13200

Team name: Moscow Aviation Inst

Number of individual grants		Cost per month	Total number of man months	Total cost (EURO)
Team Leader	1	200	15	3000
Senior Researcher	7	170	105	17850
Scientist/Engineer	0	0	0	0
Technical or Other	0	0	0	0
TOTAL				20850

Team name: Georgian Inst. Math.

Number of individual grants		Cost per month	Total number of man months	Total cost (EURO)
Team Leader	1	150	15	2250
Senior Researcher	4	130	60	7800
Scientist/Engineer	0	0	0	0
Technical or Other	0	0	0	0
TOTAL				10050

4.4.2.2 Justification Labour costs

Team 1 (Utrecht University)
*

Team 2 (Hannover University)
*

Team 3 (Marseille-Nice-Stra.)
*

Team 4 (Valladolid Univ.)
*

Team 5 (Polish Ac. Sci.)
*

Team 6 (Univ. of Liverpool)
*

Team 7 (Indep. Univ. Moscow)
*

Team 8 (Moscow State Univ.)
*

Team 9 (Steklov Inst. Math.)
*

Team 10 (Moscow Aviation Inst)
*

Team 11 (Georgian Inst. Math.)
*

4.4.2.3 Justification Operational costs

Team 1 (Utrecht University)
*

Team 2 (Hannover University)
*

Team 3 (Marseille-Nice-Stra.)
*

Team 4 (Valladolid Univ.)
*

Team 5 (Polish Ac. Sci.)
*

Team 6 (Univ. of Liverpool)
*

Team 7 (Indep. Univ. Moscow)
*

Team 8 (Moscow State Univ.)
*

Team 9 (Steklov Inst. Math.)
*

Team 10 (Moscow Aviation Inst)
*

Team 11 (Georgian Inst. Math.)
*

4.4.2.4 Justification Overheads

Team 1 (Utrecht University)
*

Team 2 (Hannover University)
*

Team 3 (Marseille-Nice-Stra.)
*

Team 4 (Valladolid Univ.)
*

Team 5 (Polish Ac. Sci.)
*

Team 6 (Univ. of Liverpool)
*

Team 7 (Indep. Univ. Moscow)
*

Team 8 (Moscow State Univ.)
*

Team 9 (Steklov Inst. Math.)
*

Team 10 (Moscow Aviation Inst)
*

Team 11 (Georgian Inst. Math.)
*

4.4.2.5 Comments

Team 1 (Utrecht University)
*

Team 2 (Hannover University)
*

Team 3 (Marseille-Nice-Stra.)
*

Team 4 (Valladolid Univ.)
*

Team 5 (Polish Ac. Sci.)
*

Team 6 (Univ. of Liverpool)
*

Team 7 (Indep. Univ. Moscow)
*

Team 8 (Moscow State Univ.)
*

Team 9 (Steklov Inst. Math.)
*

Team 10 (Moscow Aviation Inst)
*

Team 11 (Georgian Inst. Math.)
*

4.5 Project innovation potential and dissemination of results

The dissemination of results will be through publications meetings and schools. The results of the Project will be published in the central mathematical press in international and Russian journals. The previewed results of the basic research can be used in a number of problems of the Singularity Theory itself, in algebraic geometry, in topology, in geometry, including the symplectic one, in ordinary and partial derivative differential equations, in calculus of variations, in control theory, and especially in mechanics and mathematical physics, where singularity theory has a great potential.
There exists a tradition in meetings and schools on singularity theory in international centers as Warsaw, Luminy, Oberwolfach, Suszdal, Cambridge, Sao Carlos. In cooperations with these centers (but not exclusive) we intend to organise our meetings and schools for youngsters. At least one meeting will be on the interface between singularity theory and mathematical physics.
Some of the outputs of the project are implementations of various algorithms. These will be free available for the scientific community.
The public publicity of singularity theory and applications is an important topic. We intend to join national and international actions on the awareness of mathematics. A special goal is to interest potential students for mathematics through singularity theory (e.g. by master classes or guest teaching at schools).