Exercises Virasoro algebras and positive energy representations

Deadline: Friday June 6

May 13, 2008

You are allowed to work on and to hand in these exercises in groups of two persons. Oral exams are on Thursday June 12 and Friday June 13, on the following times:

10:00–10:30
10:45–11:15
11:30–12:00
13:00–13:30
13:45–14:15
14:30–15:00
15:15–15:45
16:00–16:30 (This last one can only be chosen if the others are occupied)

Send an e-mail to J.W.vandeLeur@uu.nl with the time slot you prefer most, also give a second and third choice. Time slots are distributed on a first come, first served basis.

Exercise 1. Show that $h^{p(n-1)}$, where $p(n-1)$ is the number of partitions of $n-1$, divides $\det_n(c,h)$ (without using the explicit formula (8.12)).

Exercise 2. Show that $L(c,0)$ is not unitary for $c < 1$, except possibly the points

$$c_m = c(m) = 1 - \frac{6}{(m+2)(m+3)}, \qquad \text{with } m \in \mathbb{Z}_+ \,.$$

Now, you may use formula (8.12).

Exercise 3. Prove the *Theorem of Cartan* for $\hat{sl}_2$ and $\lambda_1, \lambda_2 \in P_+$:

$$[L(\lambda_1) \otimes L(\lambda_2) : L(\lambda_1 + \lambda_2)] = 1 \,,$$

i.e., prove that if both $L(\lambda_i)$ are unitary, then $L(\lambda_1 + \lambda_2)$ occurs with multiplicity 1 in $L(\lambda_1) \otimes L(\lambda_2)$.

Exercise 4. Which "unitary points" $(c_m, h_{r,s}(m))$ can be seen to be unitary by using the Theorem of Cartan, without using the theta function calculations and results from *Lecture 11*?