Cardano 1545: Vergelijking $x^3+px+q=0$ heeft opl. (o.a.)

$$x=\sqrt[3]{-\frac{q}{2}+\sqrt{\left(\frac{q}{2}\right)^2+\left(\frac{p}{3}\right)^3}}+\sqrt[3]{-\frac{q}{2}-\sqrt{\left(\frac{q}{2}\right)^2+\left(\frac{p}{3}\right)^3}}$$

niet voor tentamen hoor!

Voorbeeld:

$x^3+6x-20=0$ hier: $\frac{q}{2}=-10$, $\frac{p}{3}=2$

dus $x=\sqrt[3]{10+\sqrt{108}}+\sqrt[3]{10-\sqrt{108}}$

Merk op: $(1\pm\sqrt{3})^3=1\pm3\sqrt{3}+9\pm3\sqrt{3}=10\pm6\sqrt{3}=10\pm\sqrt{108}$

$x=(1+\sqrt{3})+(1-\sqrt{3})=2$

Klad.

Driehoek v. Pascal

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a+b)^6 = a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6$$

<u>Voorbeeld 2</u> $x^3 - 15x - 4 = 0$ $\frac{q}{2} = -2$ $\frac{p}{3} = -5$

$$x = \sqrt[3]{2 + \sqrt{-121}} + \sqrt[3]{2 - \sqrt{-121}} = \sqrt[3]{2 + 11\sqrt{-1}} + \sqrt[3]{2 - 11\sqrt{-1}}$$

Merk op: $(2 \pm \sqrt{-1})^3 = 8 \pm 12\sqrt{-1} - 6 \mp \sqrt{-1} = 2 \pm 11\sqrt{-1}$

Maw: $x = (2 + \sqrt{-1}) + (2 - \sqrt{-1}) = 4$

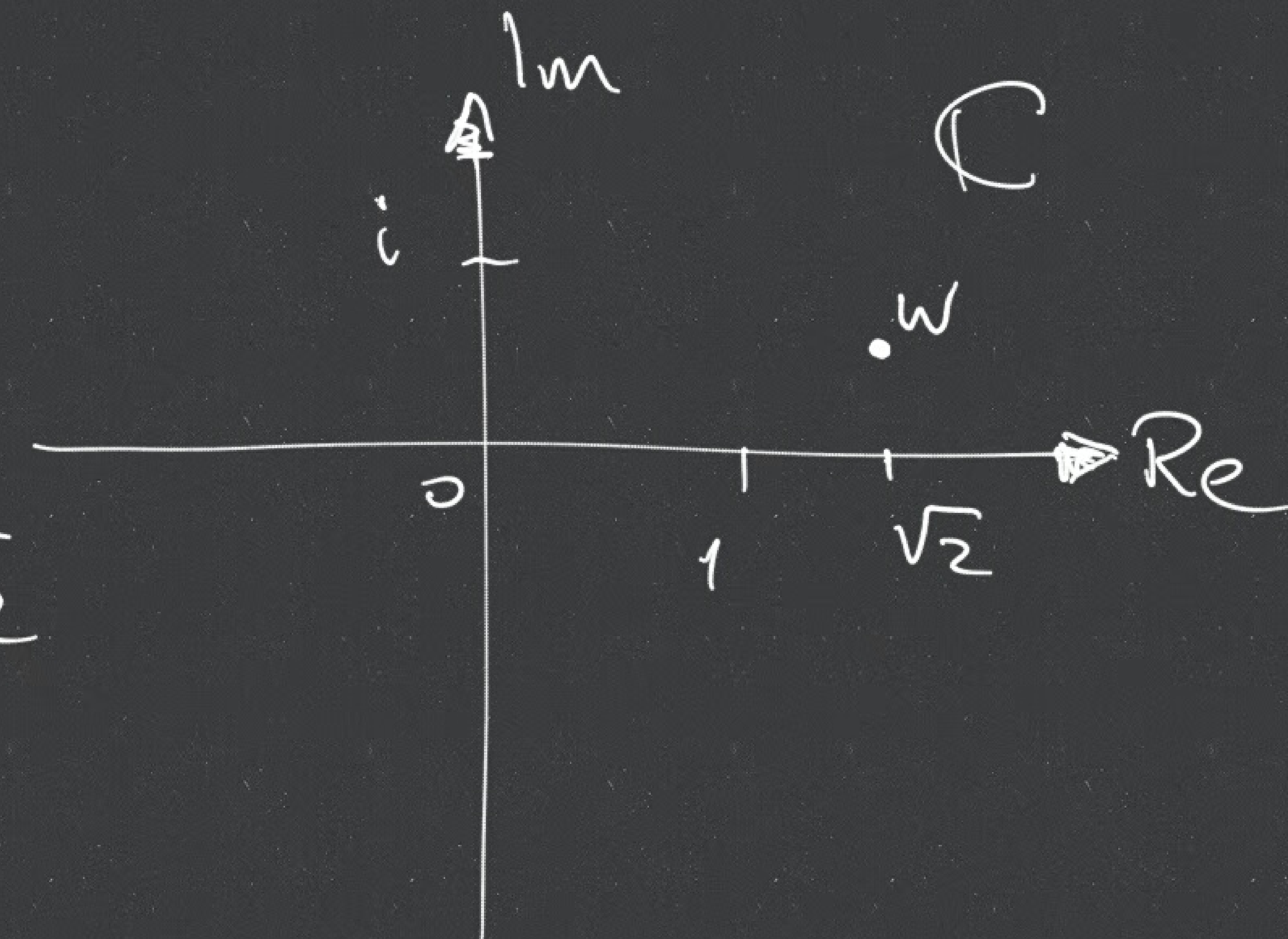
<u>NB 1</u> Necessity is the mother of invention!

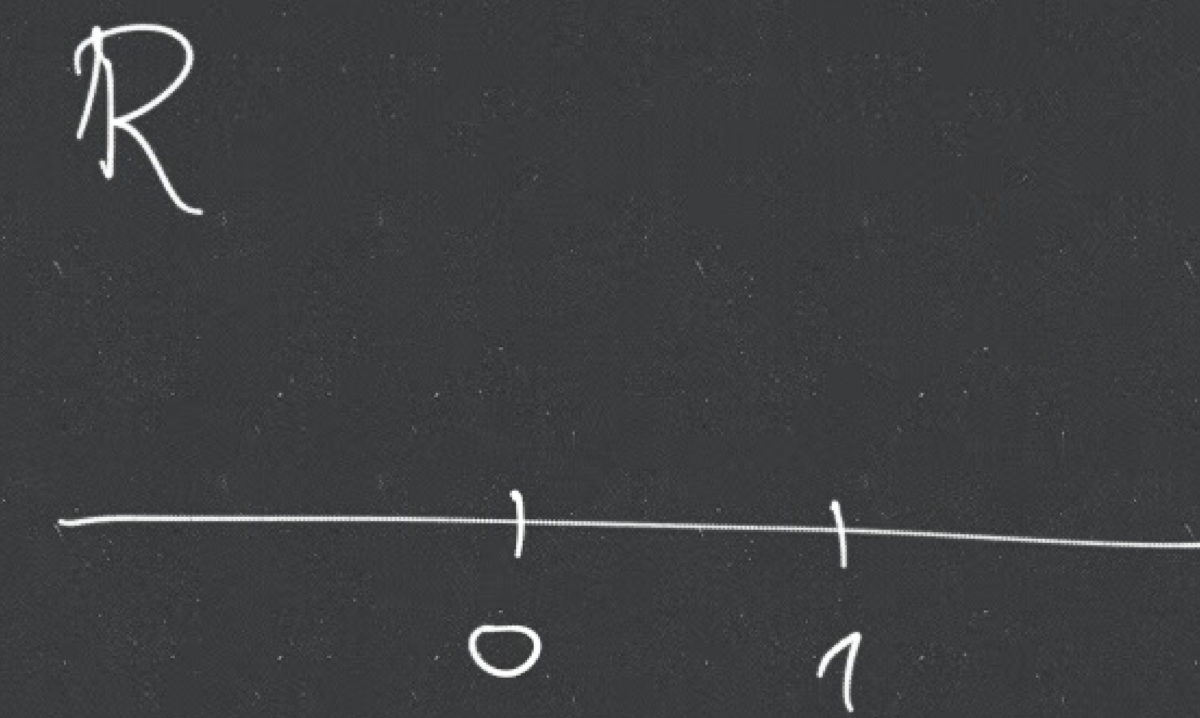
<u>NB 2</u>: $i^2 = -1$ $\longrightarrow$ Complexe getallen $\mathbb{C}$

$z = x + iy$ vgl: $\mathbb{R}$

$z = x + iy$

$w = \sqrt{2} + \frac{1}{2} i$





$\mathrm{Re}(z) = x \qquad \mathrm{Re}(w) = \sqrt{2}$

$\mathrm{Im}(z) = y \qquad \mathrm{Im}(w) = \frac{1}{2}$

$+, -, \times$: niks bijzonders ⊗ zo?

Delen: $\frac{w}{z} = \frac{\sqrt{2} + \frac{1}{2} i}{x + iy} \cdot \frac{x - iy}{x - iy}$ creatief Nietsdoen

$= \frac{\sqrt{2}x + \frac{1}{2}y + i\left(\frac{1}{2}x - \sqrt{2}y\right)}{x^2 + y^2}$

Merkw produt

$(a+b)(a-b) = a^2 - b^2$

Complex Conjugeren: als $z = x + iy$ dan is $\bar{z} = x - iy$ de geconjugeerde

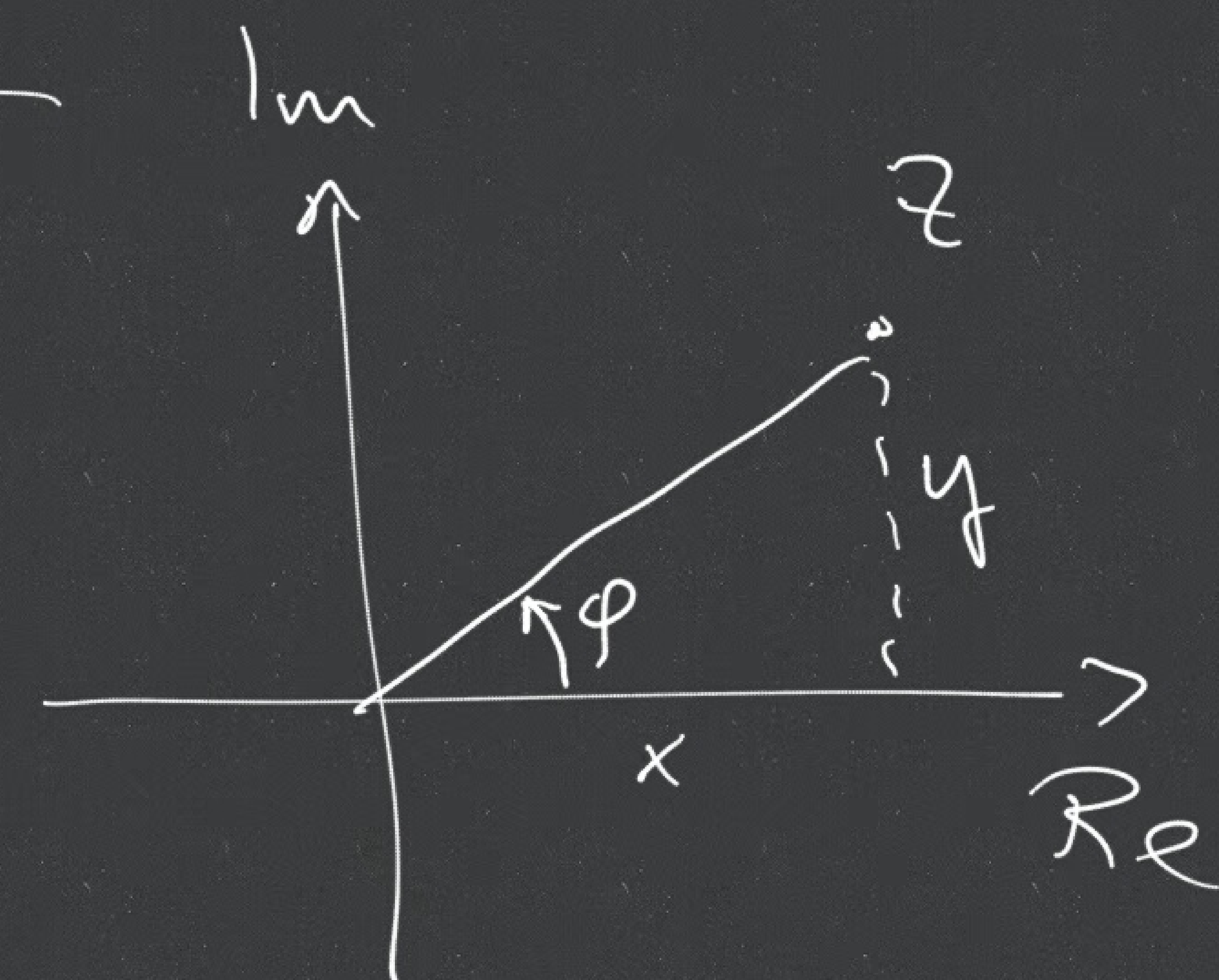
⊗ Vermenigvuldigen

$$(a+bi)(x+yi) = ax + ayi + bxi \overset{-}{\cancel{+}} by\cancel{i^2}$$

$$= ax - by + i(ay+bx)$$

Modulus van z oftewel de "lengte"

Als $z = x+iy$ dan $|z| = \sqrt{x^2+y^2}$



Argument van z dat is de hoek φ

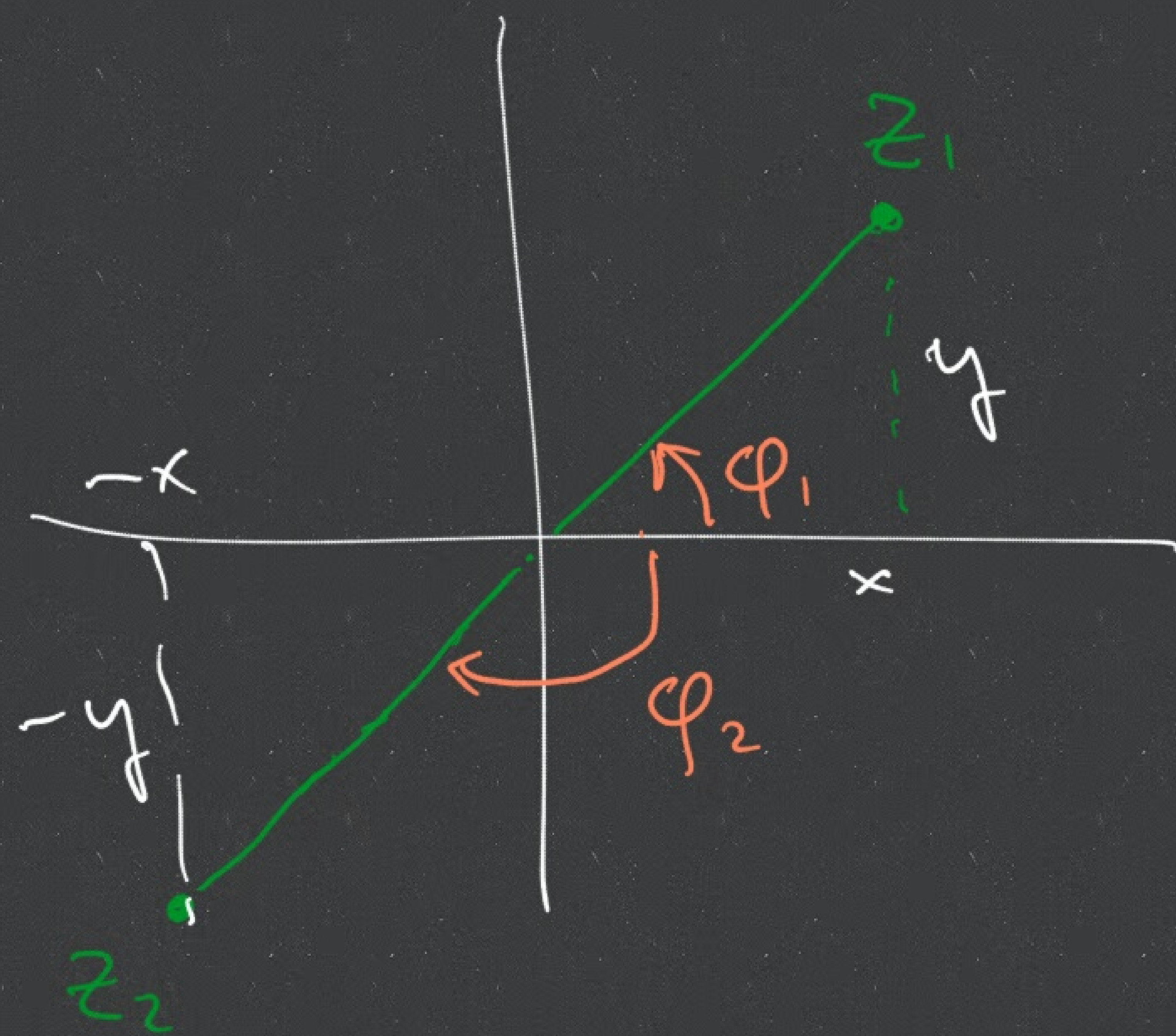
$$\sin(\arg(z)) = \frac{y}{|z|} \qquad \cos(\arg z) = \frac{x}{|z|}$$

$$\tan(\arg z) = y/x$$

dus $\arg z = \arctan \frac{y}{x} = \tan^{-1} \frac{y}{x}$

(EUR) (USA)

⚠ Gevaar!



$$\arg(z_1) = \varphi_1$$

$$\arg(z_2) = \varphi_2$$

$$\varphi_1 \neq \varphi_2$$

$$\tan\varphi_1 = \tan\varphi_2 \quad ↯$$

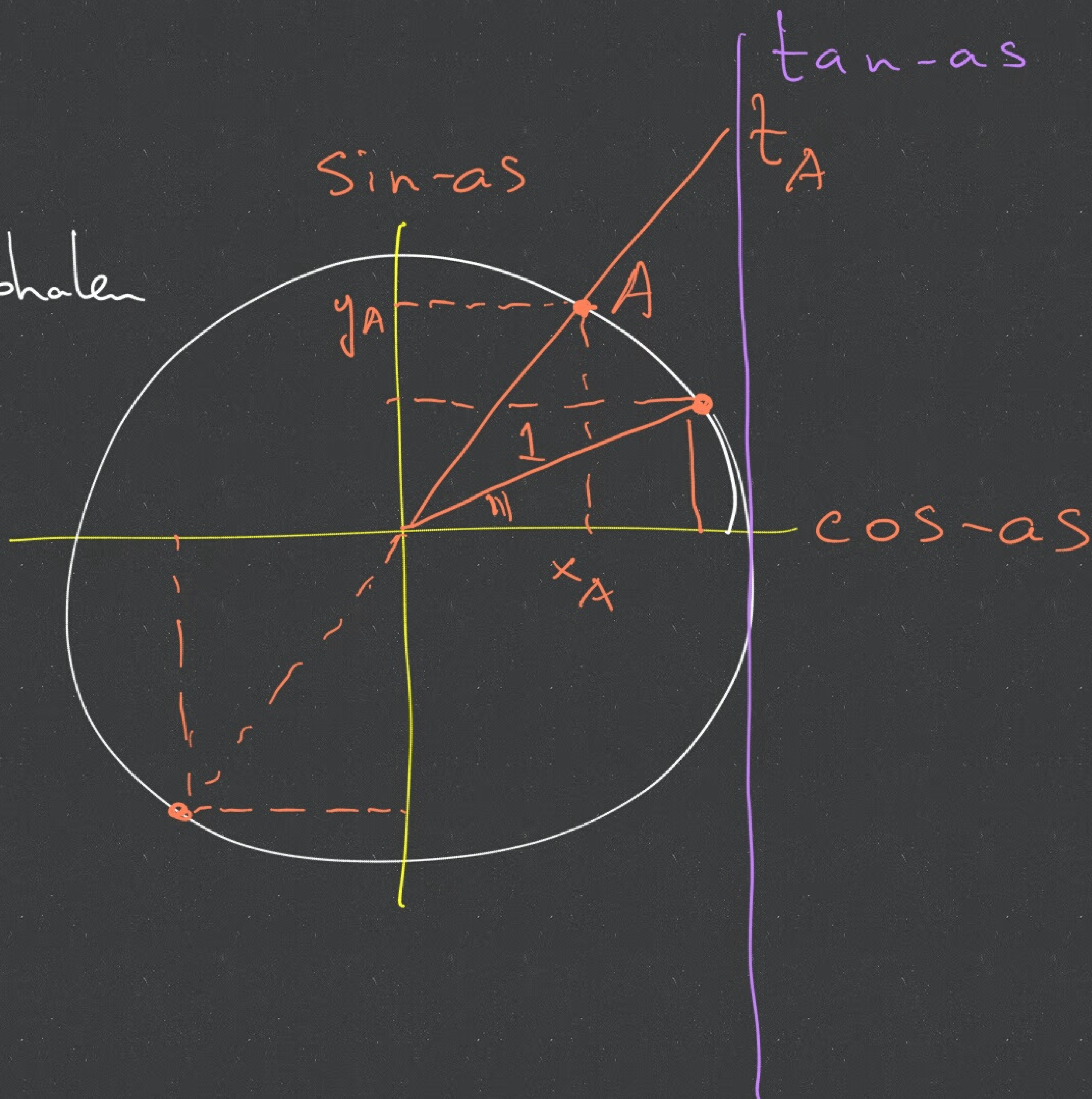
$$\tan\varphi_1 = \frac{y}{x}$$

$$\tan\varphi_2 = \frac{-y}{-x} = \frac{y}{x} = \tan\varphi_1$$

MAP:
maak altijd plaatje!

KLAD.

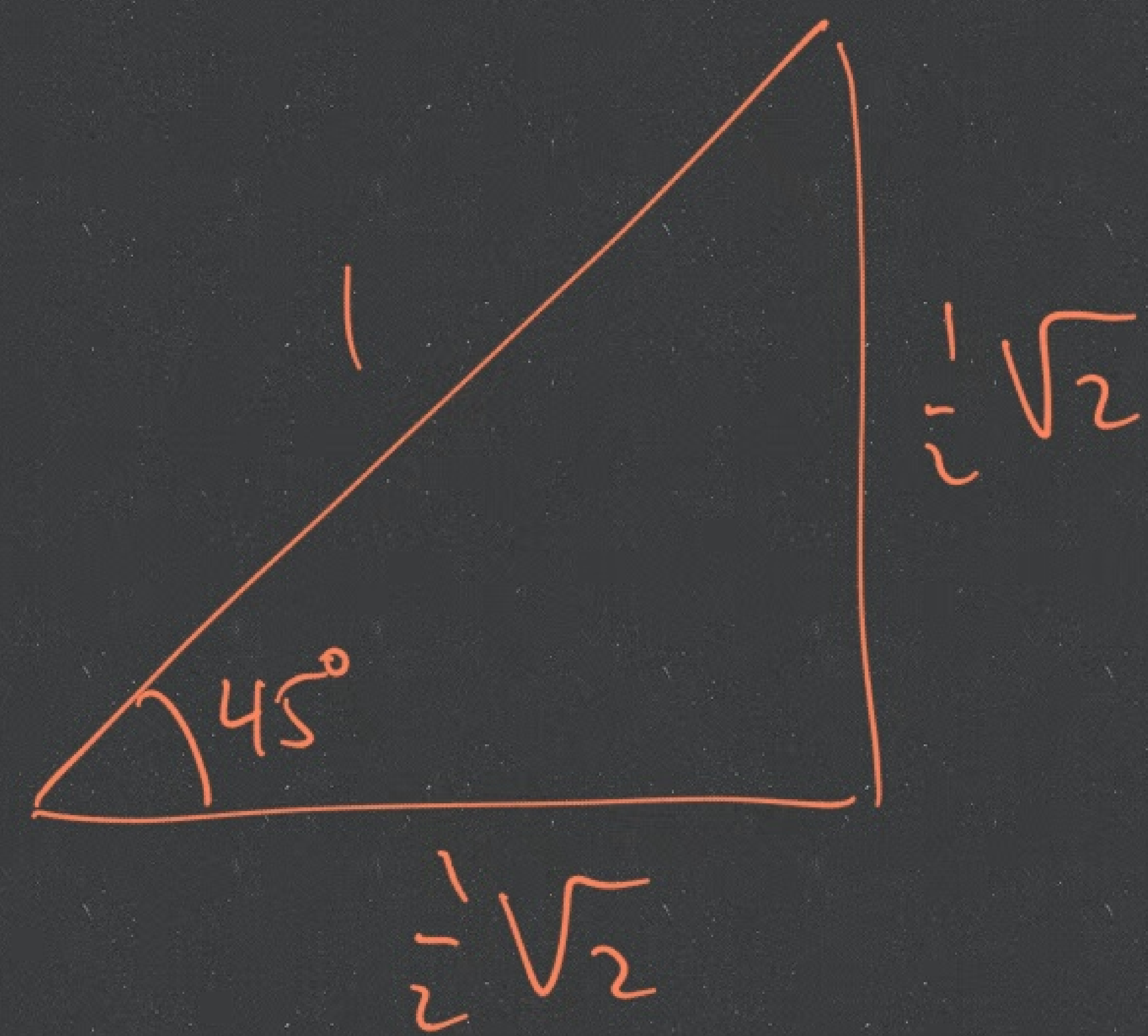
Even gonio ophalen



$\sin A = y_A$

$\cos A = x_A$

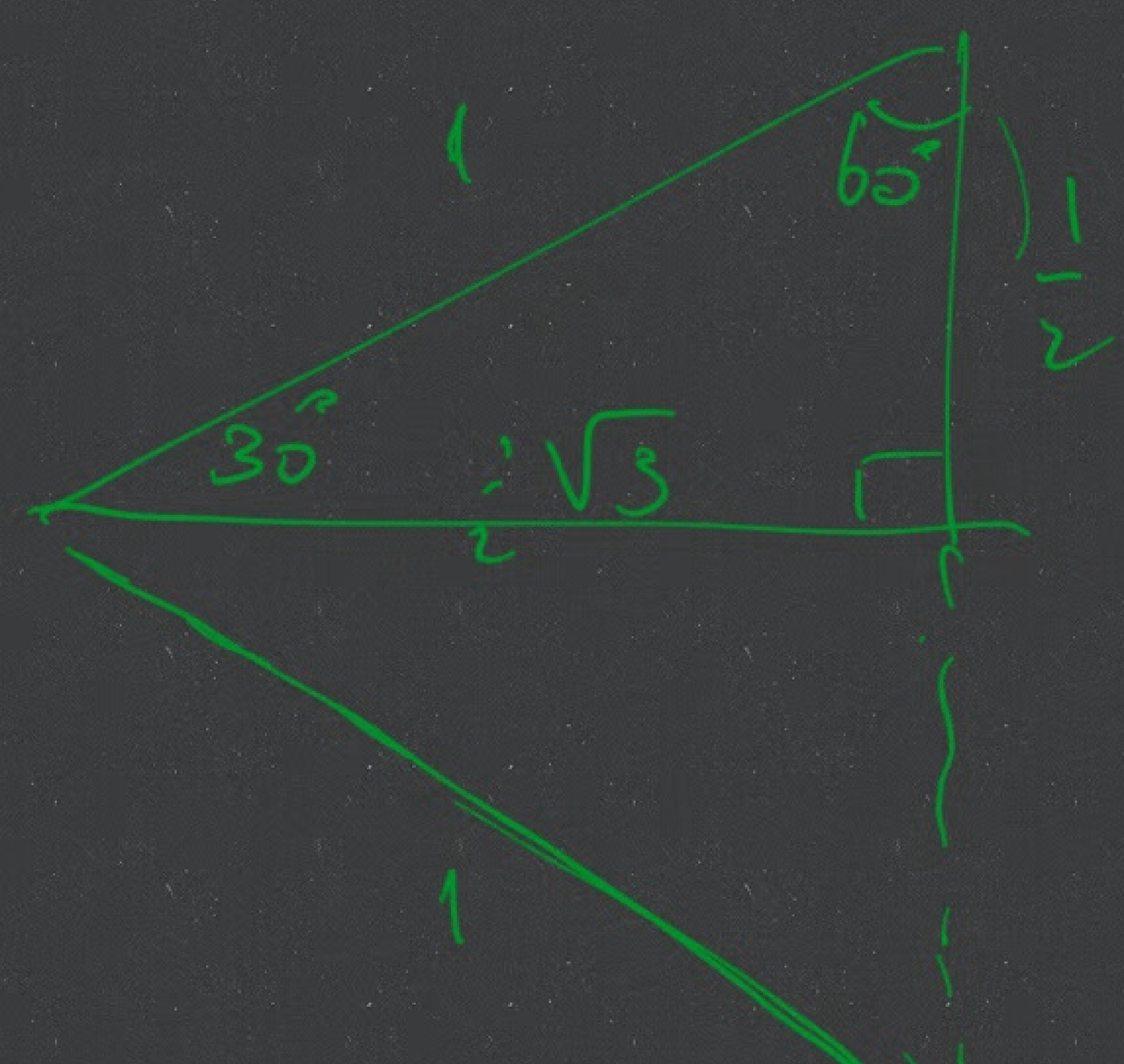
$\tan A = \frac{y_A}{x_A} = \frac{t_A}{1}$



$$45° = \frac{\pi}{4}$$

$$\cos\frac{\pi}{4} = \frac{1}{2}\sqrt{2}$$

$$\sin\frac{\pi}{4} = \frac{1}{2}\sqrt{2}$$



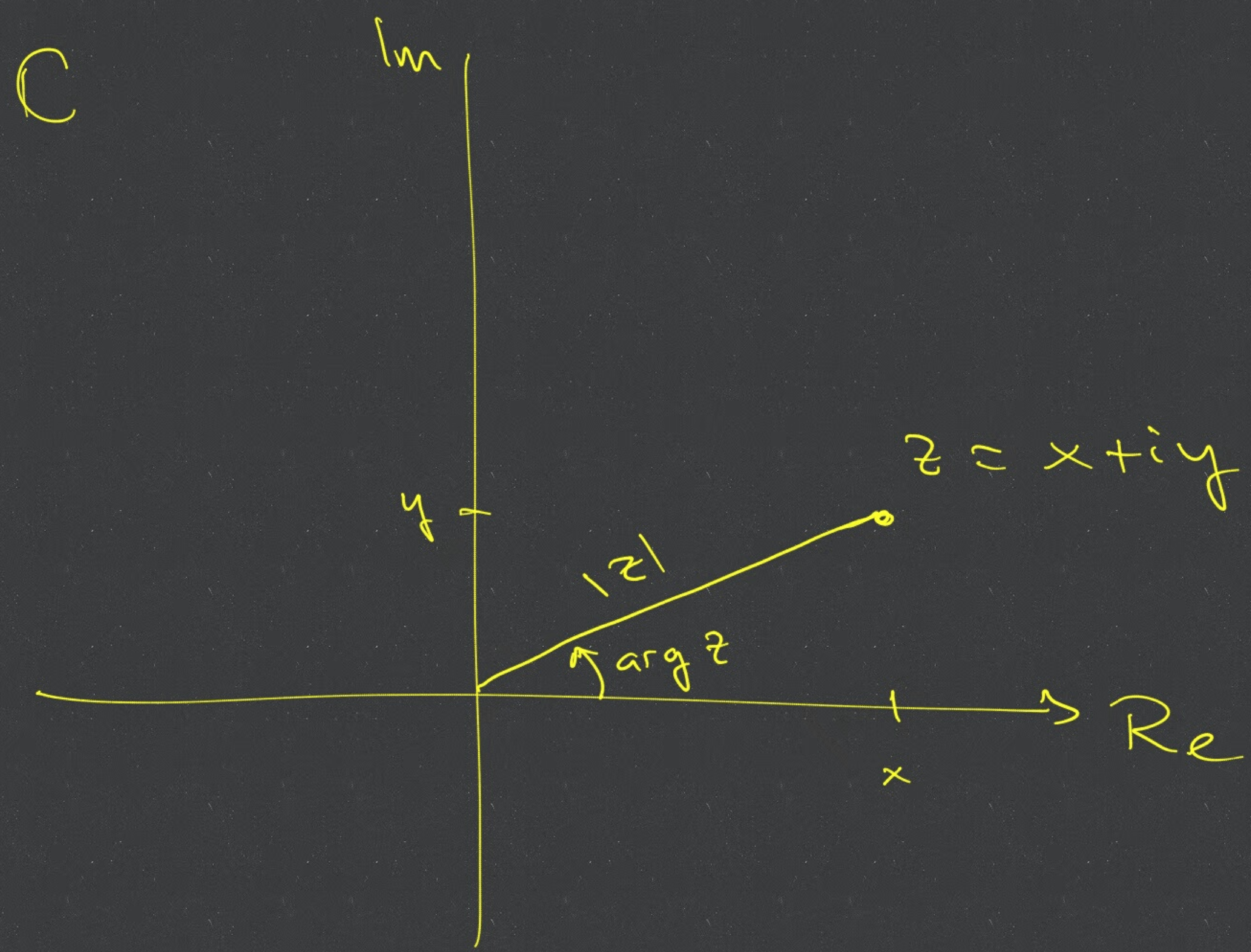
$$\cos\frac{\pi}{3} = \sin\frac{\pi}{6} = \frac{1}{2}$$

$$\sin\frac{\pi}{3} = \cos\frac{\pi}{6} = \frac{1}{2}\sqrt{3}$$

$$30° = \frac{\pi}{6}$$

$$60° = \frac{\pi}{3}$$

$$180° = \pi$$



"Rechthoeks"- voorstelling

$\mathrm{Re}(z) = x = |z| \cos(\arg z)$

$\mathrm{Im}(z) = y = |z| \sin(\arg z)$

Poolvoorstelling

$|z| = \sqrt{x^2 + y^2}$

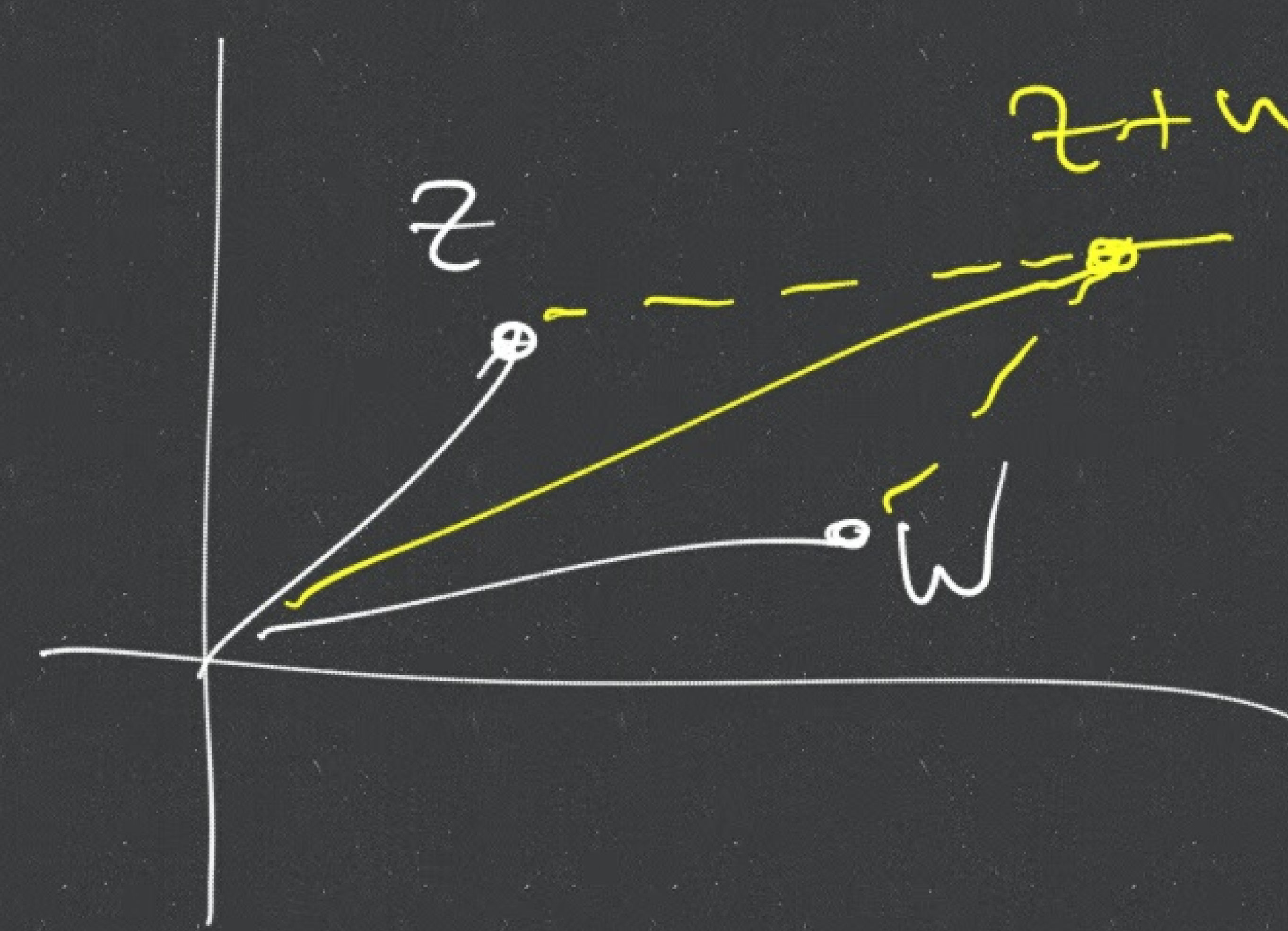
$\tan \arg z = y/x$

Rechthoeksvoorstelling: is makkelijk voor + en − :

$z = x + iy$

$w = a + ib$

$z + w = (x+a) + i(y+b)$



Poolvoorstelling is makkelijk voor vermenigvuldigen:

$|z| = r, \quad \arg z = \varphi \qquad z = r\cos\varphi + ir\sin\varphi$

$|w| = s, \quad \arg w = \psi \qquad w = s\cos\psi + is\sin\psi$

$z \cdot w = \left(r(\cos\varphi + i\sin\varphi)\right) \cdot \left(s(\cos\psi + i\sin\psi)\right) =$

$= (rs\cos\varphi\cos\psi - rs\sin\varphi\sin\psi) + i(rs\sin\varphi\cos\psi + rs\cos\varphi\sin\psi)$

$= rs\cos(\varphi+\psi) + irs\sin(\varphi+\psi)$

Conclusie:

$|zw|$ vind je door de moduli van z en w te vermenigvuldigen: $|zw| = |z|\,|w|$

$\arg(zw)$ vind je door de arg van z en w op te tellen: $\arg(zw) = \arg z + \arg w$

Daarom: vermenigv. is makkelijk in poolvoorstelling