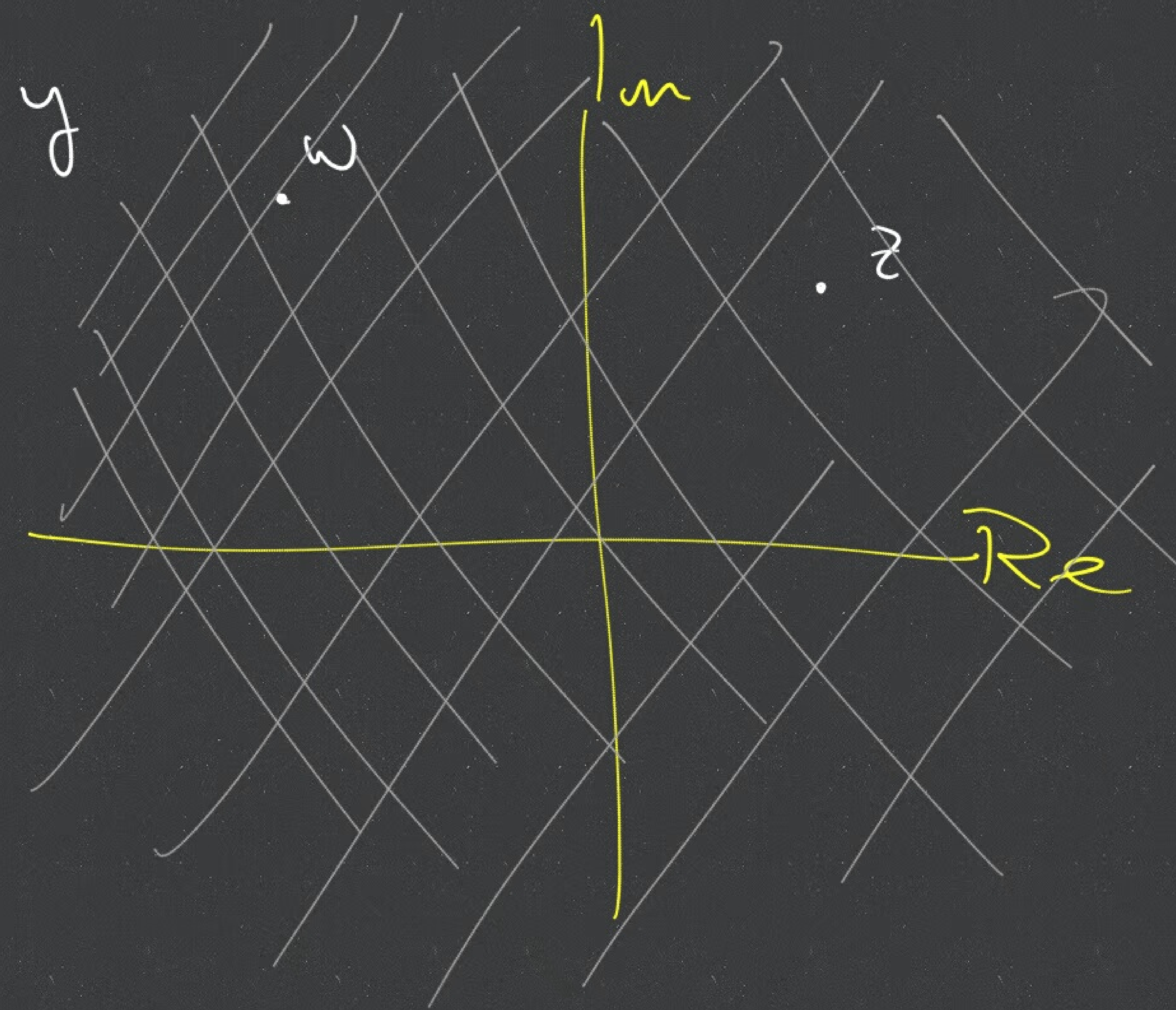
$\mathbb{C}$: $z = \mathrm{Re}(z) + \mathrm{Im}(z) \qquad = |z|(\cos \arg z + i \sin \arg z)$

! **Geen ordening:** $z, w \in \mathbb{C}$: $z = w$, $\cancel{z < w}$ of $\cancel{z > w}$

Bij $x, y \in \mathbb{R}$: $x < y$, $x = y$, $x > y$



Voorbeeld:

$z = 3 + 2i$

$w = 5 + 5i$

$q = 1 + 7i$?!?

Neem $z = x + iy$. Kijk naar $e^z = e^{x+iy} = \underline{\underline{e^x}}\,\underline{\underline{e^{iy}}}$

reëel bekend

nieuw ding

$g(x) = e^x$

$g'(x) = e^x$

Noem $f(y) = e^{iy}$. Onderzoeken:

$f(0) = 1$ ✓

Wil graag hebben:

✓ $f'(y) = (e^{iy})' = ie^{iy}$

✓ $f''(y) = -e^{iy}$

✓ $f'''(y) = -ie^{iy}$

✓ $f''''(y) = f(y)$

Herinner:

$\cos y = -\sin y$

$-\sin y = -\cos y$

$-\cos y = \sin y$

$\sin y = \cos y$

Wil ook graag hebben:

$$f(y+q) = e^{i(y+q)} = e^{iy} e^{iq} \leftarrow$$

We Definieren: $e^{iy} = \cos y + i \sin y$

Check wenselijstje:

$$e^{i0} = \cos 0 + i \sin 0 = 1$$

$$\left(e^{iy}\right)' = -\sin y + i \cos y = i(\cos y + i \sin y)$$

Neem aan: $e^{ix} = \cos x + i \sin x \quad (1)$

Neem aan: $\underline{\underline{e^{i(a+b)}}} = \underline{\underline{e^{ia} e^{ib}}}$

↓ uitschrijven mbv (1)

↓ uitschrijven mbv (1) + haakjes uitwerken

tenslotte: vergelijk L & R de reële delen
L & R de imag. delen.

$$\underbrace{\cos(a+b)}_{\text{Reeel}} + i \underbrace{\sin(a+b)}_{\text{Im.}} = (\cos a + i \sin a)(\cos b + i \sin b)$$

$$= \boxed{\cos a \cos b - \sin a \sin b}$$

$$+ i(\underline{\sin a \cos b + \cos a \sin b})$$

Conclusie

$\cos(a+b) = \cos a \cos b - \sin a \sin b$

$\sin(a+b) = \sin a \cos b + \cos a \sin b$

$\cos(a+b) = \cos a \cos b - \sin a \sin b$

$\sin(a+b) = \sin a \cos b + \cos a \sin b$

Verschilform.

$$\cos(a-b) = \cos a \cos(-b) - \sin a \sin(-b)$$
$$= \cos a \cos b + \sin a \sin b$$

$$\sin(a-b) = \sin a \cos b - \cos a \sin b$$

Verdubbeling:

$$\cos(2a) = \cos^2 a - \sin^2 a \overset{+\sin^2 a - \sin^2 a}{=} 1 - 2\sin^2 a$$
$$= 2\cos^2 a - 1$$

$$\sin(2a) = 2 \sin a \cos a$$

Bereken $e^{i\pi/2} = \cos\frac{\pi}{2} + i \sin\frac{\pi}{2} = i$

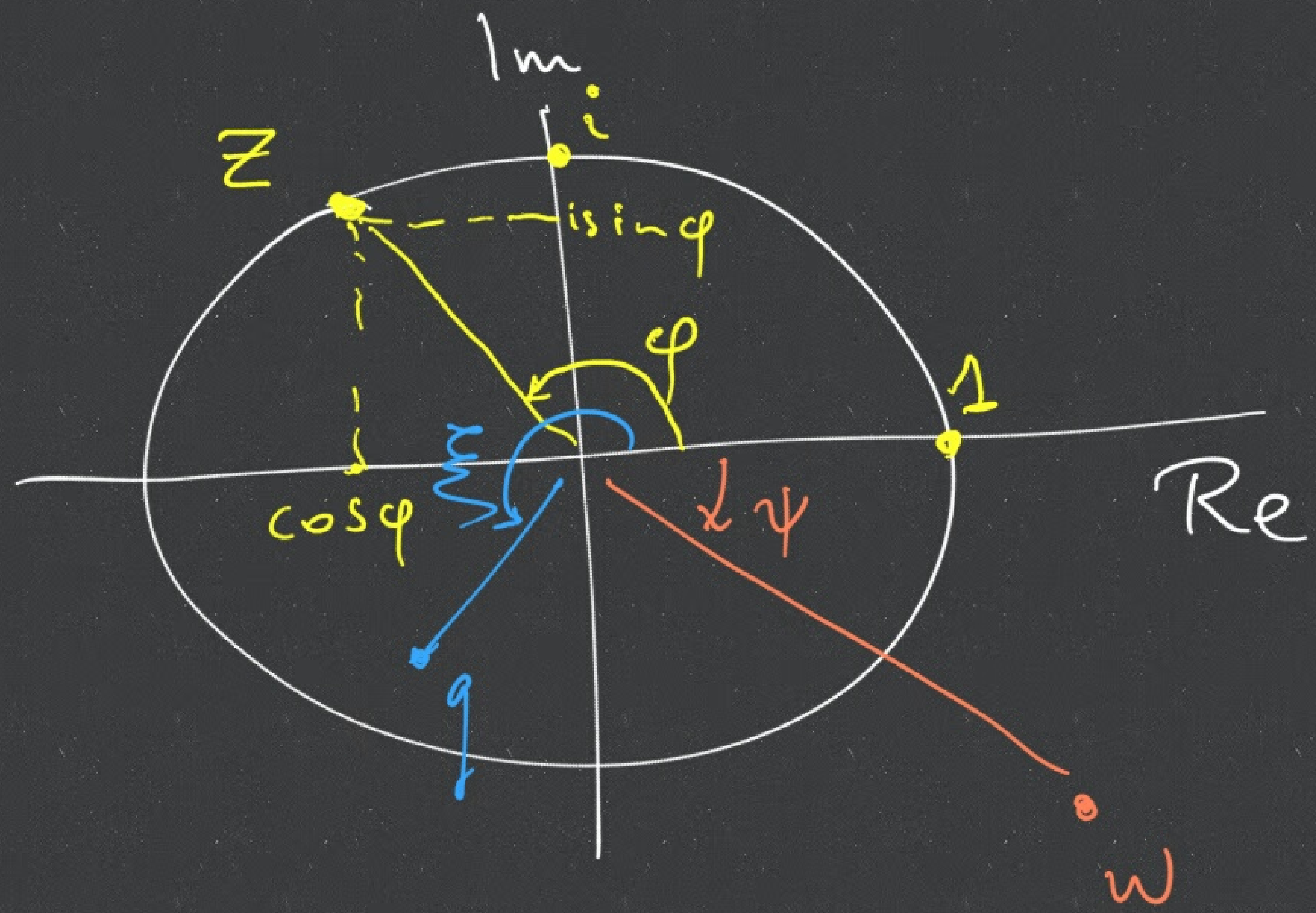
$$e^{i\pi} = \cos \pi + i \sin \pi = -1$$

De Moivre:

$$\begin{array}{ccc} e^{inx} & = & \cos nx + i \sin nx \\ \| & & \| \\ (e^{ix})^n & = & (\cos x + i \sin x)^n \end{array}$$

$$\left.\begin{aligned} e^{ix} &= \cos x + i \sin x \\ e^{-ix} &= \cos x - i \sin x \end{aligned}\right\} \rightarrow \begin{aligned} 2\cos x &= e^{ix} + e^{-ix} \\ 2i \sin x &= e^{ix} - e^{-ix} \end{aligned}$$

Hee leuk!



$$|z| = 1$$

$$\arg z = \varphi$$

$$z = 1e^{i\varphi} \qquad !!!$$

$$|w| \neq 1, \quad \arg w = \psi$$

$$w = |w|\, e^{i\psi}$$

$$|q| \neq 1, \quad \arg q = \xi$$

$$wq = |w| e^{i\psi} \cdot |q| e^{i\xi}$$

$$= \underbrace{|w||q|}_{\text{product vd mod}}\, e^{i(\underbrace{\psi+\xi}_{\text{som vd arg}})}$$

$$\frac{w}{q} = \frac{|w|}{|q|} e^{i(\psi-\xi)}$$

$$w^2 = |w|^2 e^{2i\psi}$$

$$w^n = |w|^n e^{n\psi i}$$

$$\sqrt{w} = w^{\frac{1}{2}} = \sqrt{|w|}\, e^{i\psi/2} \text{ of } \sqrt{|w|}\, e^{i\left(\frac{\psi}{2}+\pi\right)}$$

NB: $|w|e^{i\psi} = |w|e^{i(\psi+2\pi)}$ zijn hetzelfde getal!

"<u>De</u>" wortel van w kan niet meer.

$\sqrt{|w|}\,e^{i(\frac{\psi}{2}+\pi)}$

$\sqrt{|w|}\,e^{i\psi/2}$ Hoofdwaarde van $\sqrt{w}$

Is ook een wortel van w

$e^{i\,7\pi/3}$

$$w^{\frac{1}{n}} = |w|^{\frac{1}{n}}\, e^{i\left(\frac{\psi}{n}+\frac{2k\pi}{n}\right)}$$ met $k = 0, 1, 2, \ldots, n-1$