

"Inverse Subs" met name gonio-subst (hyp. subst over slaan)

3 situaties

① bij integralen met $\sqrt{a^2 - x^2}$: denk aan $x = \sin u$

$$\text{Vb. } \int \frac{dx}{\sqrt{1-x^2}} = \int \frac{\cos u \, du}{\sqrt{1-\sin^2 u}} \quad dx = \cos u \, du$$

$$= \int \frac{\cos u}{\cos u} \, du = \int du = u + c$$

$$= \arcsin x + c$$

$$\text{Vb. } \int \sqrt{5-x^2} \, dx = \sqrt{5} \int \sqrt{1-\frac{x^2}{5}} \, dx \quad \left\{ \begin{array}{l} \text{kies } \frac{x}{\sqrt{5}} = \sin u \\ \text{of } x = \sqrt{5} \sin u \end{array} \right.$$
$$= \sqrt{5} \int \sqrt{1-\sin^2 u} \, \sqrt{5} \cos u \, du \quad \left\{ \begin{array}{l} dx = \sqrt{5} \cos u \, du \end{array} \right.$$

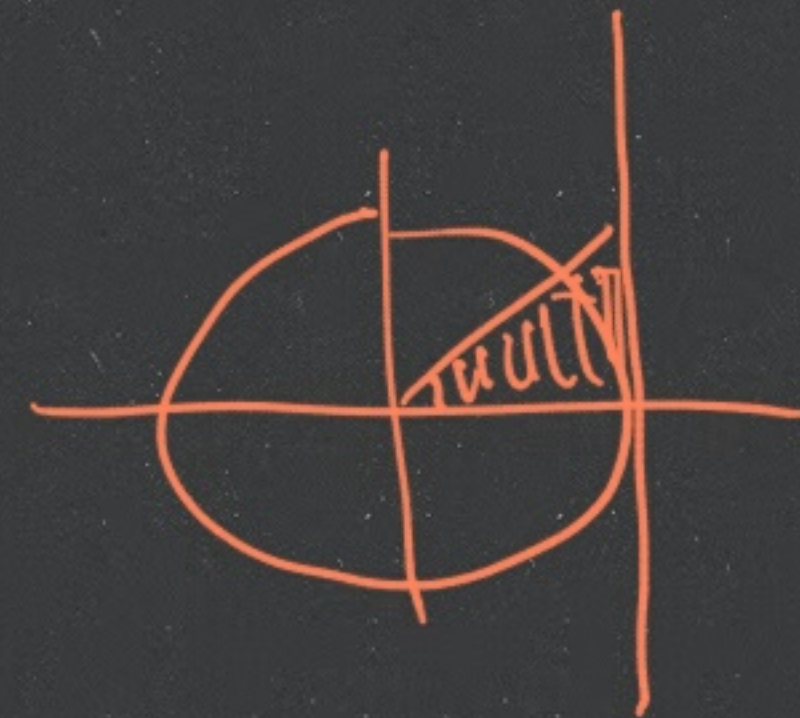
$$= 5 \int \cos^2 u \, du$$

Verdubbelingsform

$$\text{of } \begin{array}{l} \cos 2x = 2\cos^2 x - 1 \\ \cos^2 x = \frac{1}{2}(\cos 2x + 1) \text{ ETC} \end{array}$$

② Bij $\sqrt{x^2 - a^2}$ denken aan $x = \frac{1}{\cos u} = \sec u$ [som's werkt]
 $x = \cosh u$

Vb $\int \frac{dx}{\sqrt{x^2 - 1}}$



$dx = + \frac{\sin u}{\cos^2 u} du$ invullen

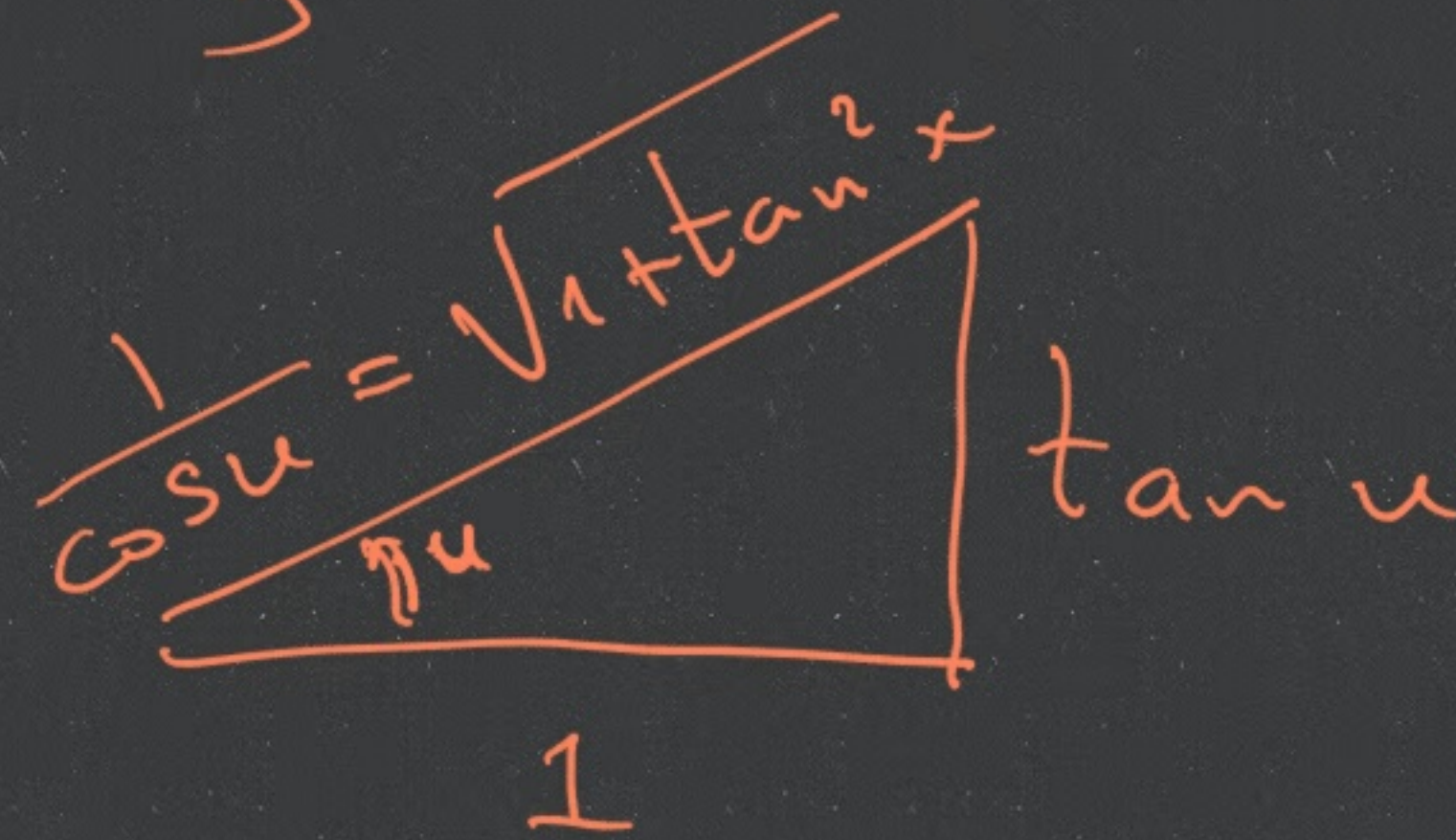
$= \int \sqrt{\frac{\cos^2 u}{1 - \cos^2 u}} \cdot \frac{\sin u}{\cos^2 u} du$

$= \int \frac{\cos u}{\sin u} \cdot \frac{\sin u}{\cos^2 u} du$

$= \int \frac{1}{\cos u} du$

$= \log \left| \frac{1}{\cos u} + \tan u \right| + c$

STANDAARD-INT



dus $\tan^2 u = \frac{1}{\cos^2 u} - 1$

$x^2 - 1 = \frac{1}{\cos^2 u} - 1 =$
 $= \frac{1}{\cos^2 u} - \frac{\cos^2 u}{\cos^2 u}$
 $= \frac{1 - \cos^2 u}{\cos^2 u}$

dus $\frac{1}{\sqrt{x^2 - 1}} = \sqrt{\frac{\cos^2 u}{1 - \cos^2 u}}$

$\log \left| x + \sqrt{x^2 - 1} \right|$

Werkst
ooh:

$$\tan u = \frac{\sin u}{\cos u}$$

$$\frac{1}{\cos u} = x$$

$$\sin u = \sqrt{1 - \frac{1}{x^2}}$$

$$\tan u = x \sqrt{1 - \frac{1}{x^2}}$$

$$= \sqrt{x^2 - 1}$$

Hoeverd uur incl. hc + wc
heb je afgelopen week = 7 dg.
aan wistech besteed?

6

9

③ Bij a^2+x^2 : denk aan $x = \tan u$

$$\underline{Vb} \int \frac{dx}{\sqrt{4+x^2}} = \frac{1}{2} \int \frac{dx}{\sqrt{1+(\frac{x}{2})^2}}$$

$$\frac{x}{2} = \tan u, \quad x = 2 \tan u$$

$$dx = 2 \frac{du}{\cos^2 u} = 2(1+\tan^2 u) du$$

$$\frac{1}{2} \int \frac{2(1+\tan^2 u) du}{\sqrt{1+\tan^2 u}} = \int \sqrt{1+\tan^2 u} du$$

$$= \int \sqrt{\frac{1}{\cos^2 u}} du = \int \frac{1}{\cos u} du$$

$$= \log \left| \frac{1}{\cos u} + \tan u \right| + c$$

en terugsubs.

$$= \log \left| \frac{1}{\cos u} + \tan u \right| + c$$

in terms of x :

$$\log \left| \sqrt{1 + \tan^2 u} + \tan u \right| + c$$

$$= \log \left| \sqrt{1 + \left(\frac{x}{2}\right)^2} + \frac{x}{2} \right| + c$$

$$x = 2 \tan u$$

$$1 + \tan^2 u = \frac{1}{\cos^2 u}$$

$$\frac{1}{\cos u} = \sqrt{1 + \tan^2 u}$$