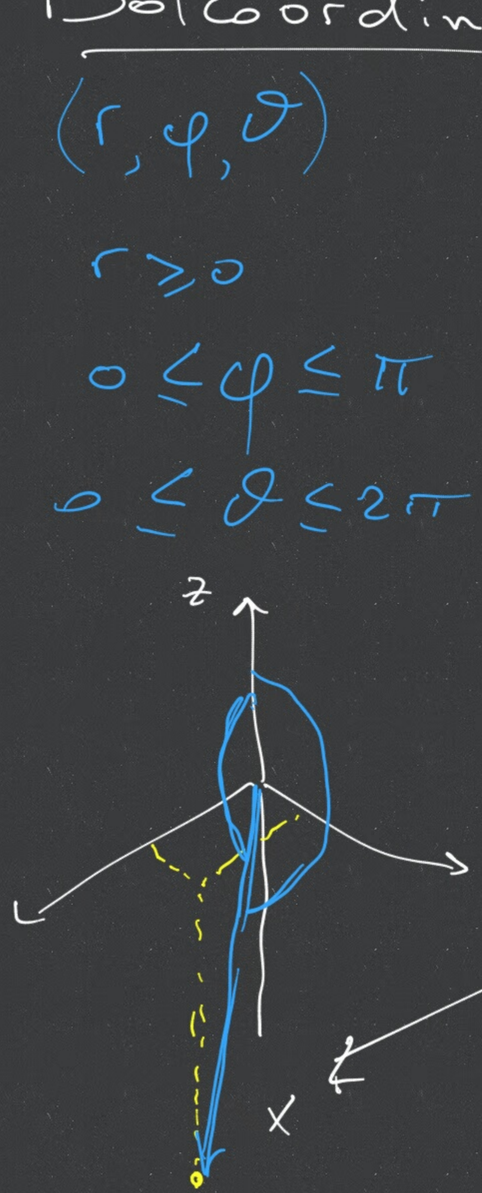
Polarkoordinaten

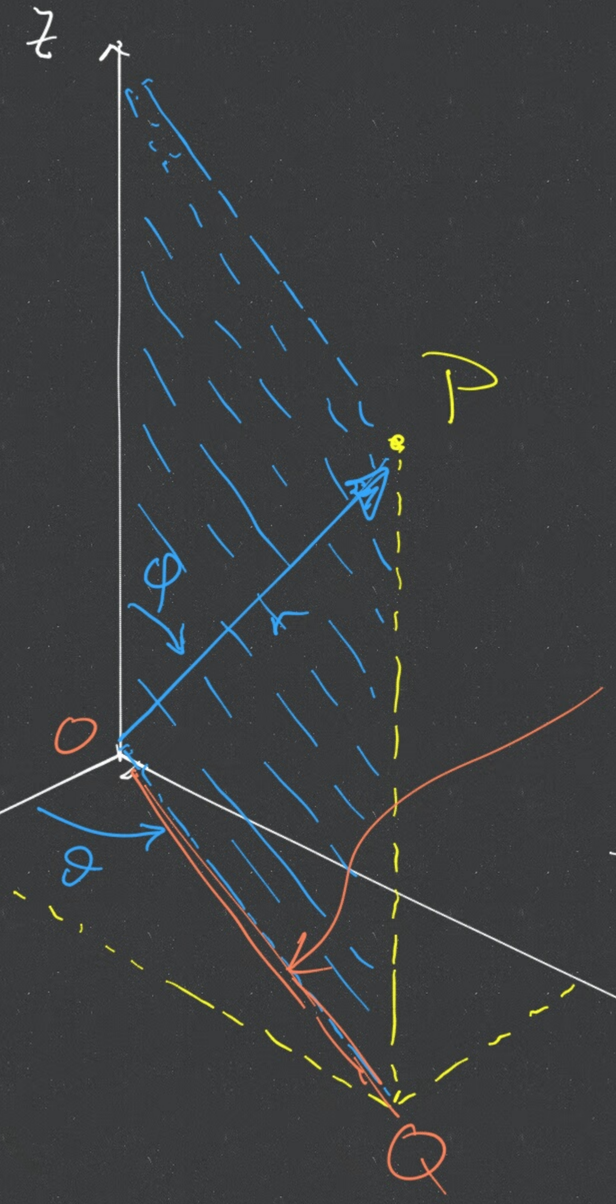
(r, φ, ϑ)

$r \geq 0$ $\qquad OP = r$

$0 \leq \varphi \leq \pi$

$0 \leq \vartheta \leq 2\pi$





Umrechnen:

$OQ = r \sin\varphi$

$PQ = r \cos\varphi$

$x = r \sin\varphi \cos\vartheta$

$y = r \sin\varphi \sin\vartheta$

$z = r \cos\varphi$

Andersum:

$r = \sqrt{x^2 + y^2 + z^2}$

$\tan\vartheta = y/x$

$\cos\varphi = z / \sqrt{x^2 + y^2 + z^2}$

Hst 11 Gaat over functies $\bar{f}: \mathbb{R} \to \mathbb{R}^3$, $t \mapsto \bar{f}(t)$

NB: $\bar{f}$ is een vector.

VB: "Plaats als functie van tijd"

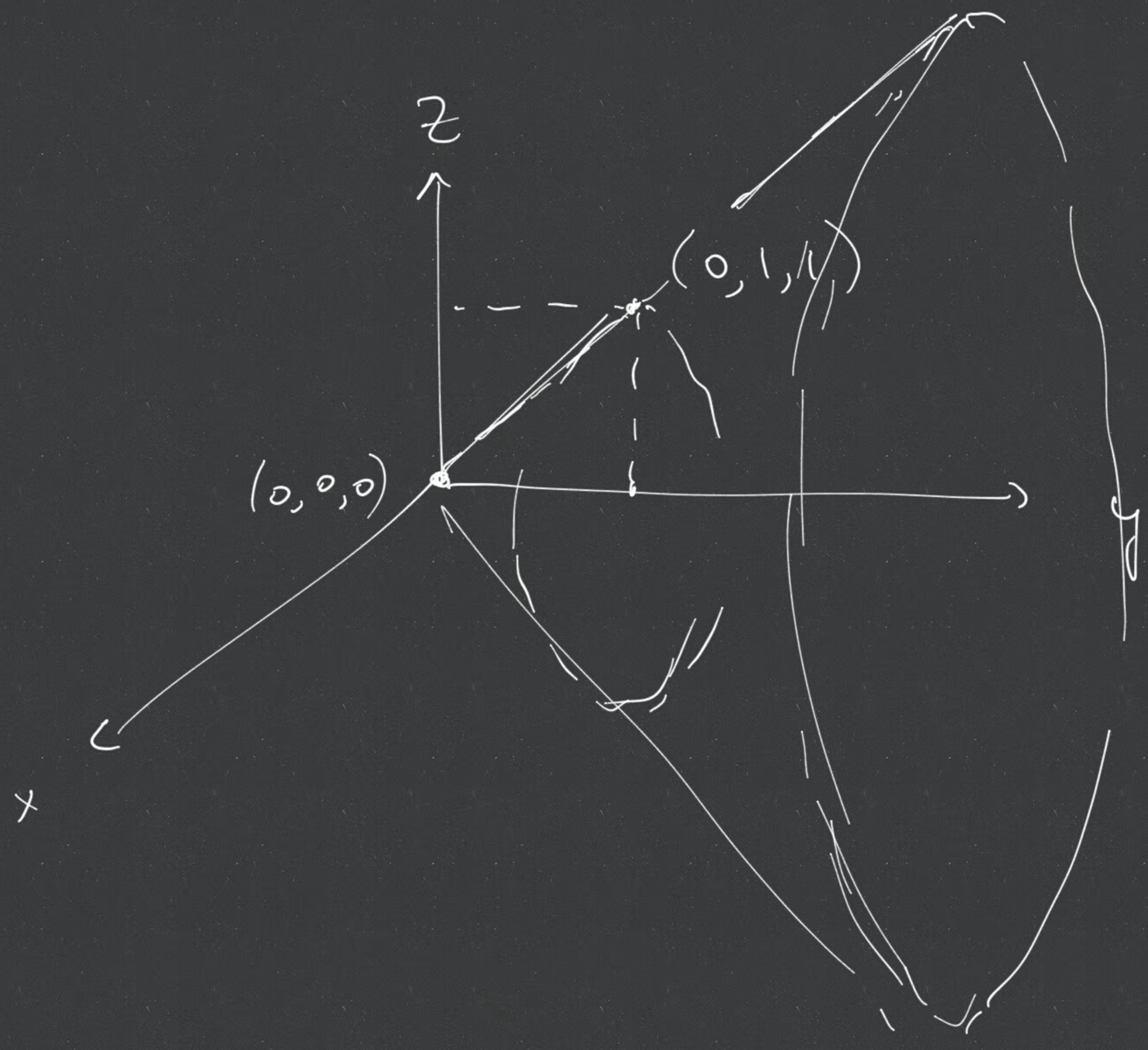
$$\bar{r}(t) = \begin{pmatrix} x(t) \\ y(t) \\ z(t) \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$ waarin $x = x(t)$, $y = y(t)$, $z = z(t)$

$$= x\hat{i} + y\hat{j} + z\hat{k}$$

Snelheid.

$$\bar{v}(t) = \frac{d}{dt}\bar{r}(t) = \dot{x}\hat{i} + \dot{y}\hat{j} + \dot{z}\hat{k} = \bar{r}'(t)$$

Engels { velocity is een vector $\bar{v}$
{ speed is een scalar $|\bar{v}|$


z
(0,1,1)
(0,0,0)
y
x

Stelling p. 627-8 Diff van vectorproducten en -sommen

in $\mathbb{R}^3$

$\bar{u}, \bar{v}$ vectoren afh. van t

λ scalair afh van t

- $\frac{d}{dt}(\bar{u}+\bar{v}) = \frac{d\bar{u}}{dt} + \frac{d\bar{v}}{dt}$
- $\frac{d}{dt}(\lambda\bar{u}) = \frac{d\lambda}{dt}\bar{u} + \lambda\frac{d\bar{u}}{dt}$
- $\frac{d}{dt}(\bar{u}\cdot\bar{v}) = \left(\frac{d}{dt}\bar{u}\right)\cdot\bar{v} + \bar{u}\cdot\frac{d\bar{v}}{dt}$ $\quad (\bar{u}\cdot\bar{v})' = \bar{u}'\cdot\bar{v} + \bar{u}\cdot\bar{v}'$
- $(\bar{u}\times\bar{v})' = \bar{u}'\times\bar{v} + \bar{u}\times\bar{v}'$

Bewijstechniek: uitschrijven in coördinaten

Bijv: $(\lambda\bar{u})' = (\lambda u_1\hat{i} + \lambda u_2\hat{j} + \lambda u_3\hat{k})'$

$= ((\lambda u_1)'\hat{i} + (\lambda u_2)'\hat{j} + (\lambda u_3)'\hat{k})$

$= ((\lambda' u_1 + \lambda u_1')\hat{i} + (\lambda' u_2 + \lambda u_2')\hat{j} + (\lambda' u_3 + \lambda u_3')\hat{k})$

$$= \left((\lambda' u_1 + \lambda u_1')\hat{i} + (\lambda' u_2 + \lambda u_2')\hat{j} + (\lambda' u_3 + \lambda u_3')\hat{k} \right)$$

$$= \left(\lambda' u_1 \hat{i} + \lambda' u_2 \hat{j} + \lambda' u_3 \hat{k} \right)$$

$$+ \left(\lambda u_1' \hat{i} + \lambda u_2' \hat{j} + \lambda u_3' \hat{k} \right)$$

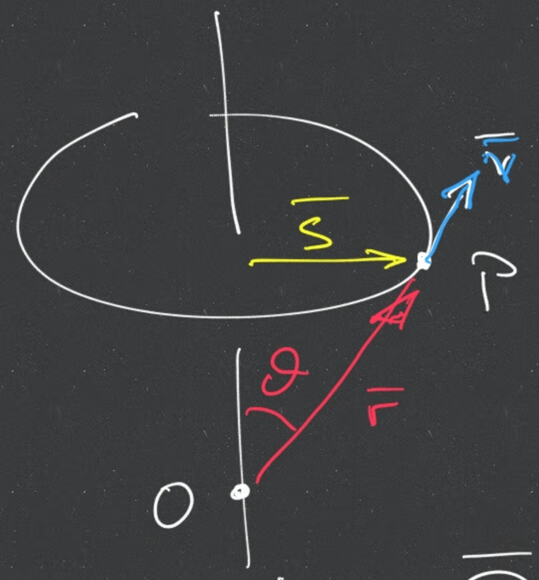
$$= \lambda' \left(u_1 \hat{i} + u_2 \hat{j} + u_3 \hat{k} \right)$$

$$+ \lambda \left(u_1' \hat{i} + u_2' \hat{j} + u_3' \hat{k} \right)$$

$$= \lambda' \vec{u} + \lambda \vec{u}'$$ ∎

Rotatie §11.2

Bekijk deeltje P in cirkelbaan.



$\bar{r}$ plaats van P

$\bar{v}$ snelheid $= \frac{d\bar{r}}{dt}$

ϑ = hoek tussen z-as en $\bar{r}$

Definieer vector $\bar{\Omega}$ door $\begin{cases} \text{richting} \parallel \text{z-as} \\ \text{grootte} = \text{hoeksnelheid van P.} \end{cases}$

$|\bar{\Omega}| \quad \frac{|\bar{v}|}{|\bar{r}|\sin\vartheta}$

Consequenties:

1) $\bar{\Omega} \perp \bar{v}$
2) $\bar{r} \perp \bar{v}$

$\Rightarrow \boxed{\bar{v} \parallel \bar{\Omega} \times \bar{r}}$

3) $|\bar{\Omega}| = \frac{|\bar{v}|}{|\bar{r}|\sin\vartheta}$ dus $\boxed{|\bar{v}| = |\bar{\Omega}||\bar{r}|\sin\vartheta = |\bar{\Omega} \times \bar{r}|}$

Conclusie:

$\bar{v} = \bar{\Omega} \times \bar{r}$