curve C in $\mathbb{R}^3$

parametriseren:

$$\bar r(t) = x(t)\,\hat i + y(t)\,\hat j + z(t)\,\hat k \qquad a \le t \le b$$

11.3 booglengte

$$\boxed{s = \int_C ds = \int_a^b \left|\frac{d\bar r}{dt}\right| dt}$$

15.3 toevoegen: scalarveld $f : \mathbb{R}^3 \to \mathbb{R}$

$$\int_C f\,ds = \int_a^b f(\bar r(t)) \left|\frac{d\bar r}{dt}\right| dt$$

15.4 met vectorveld: $\bar F : \mathbb{R}^3 \to \mathbb{R}^3$

$$\int_C \bar F \cdot d\bar s = \int_a^b \bar F(\bar r(t)) \cdot \frac{d\bar r}{dt}\, dt$$

(denk bijv. aan arbeid)

oppervlak S in $\mathbb{R}^3$

14.7 ~~Het~~ opp

De oppervlakte van het opp S
(area) (surface)

$$S = \iint_S dS$$

parametriseren $\bar r(u,v)$ met (u,v) in gebied $D \subset \mathbb{R}^2$

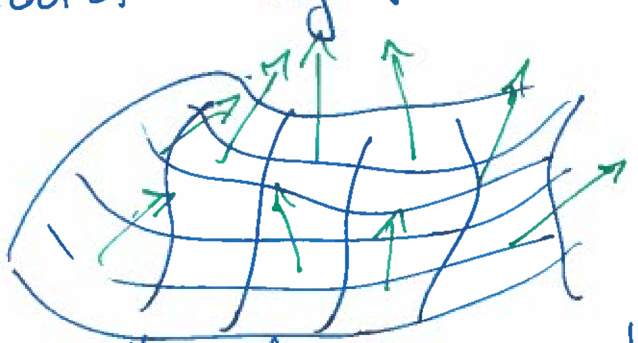
$$\boxed{S = \iint_S dS = \iint_D \left|\frac{\partial \bar r}{\partial u} \times \frac{\partial \bar r}{\partial v}\right| d(u,v)}$$

15.5 met scalarveld $f : \mathbb{R}^3 \to \mathbb{R}$

$$\iint_S f\,dS = \iint_D f(\bar r(u,v)) \left|\frac{\partial \bar r}{\partial u} \times \frac{\partial \bar r}{\partial v}\right| d(u,v)$$

Gedachte bij Flux

"doorstroming van een opp."



Je meet alleen die component van $\overline{F}$ die $\perp$ op S staat

Als je die flux integraal uitwerkt met parametrisering $\overline{r}(u,v)$ van S:

$$\iint_S \overline{F} \bullet d\overline{S} = \iint_D \overline{F}(\overline{r}(u,v)) \bullet \left(\frac{\partial \overline{r}}{\partial u} \times \frac{\partial \overline{r}}{\partial v} \right) d(u,v)$$

(15.6) met vectorveld $\overline{F}: \mathbb{R}^3 \to \mathbb{R}^3$ (2)

Definitie:

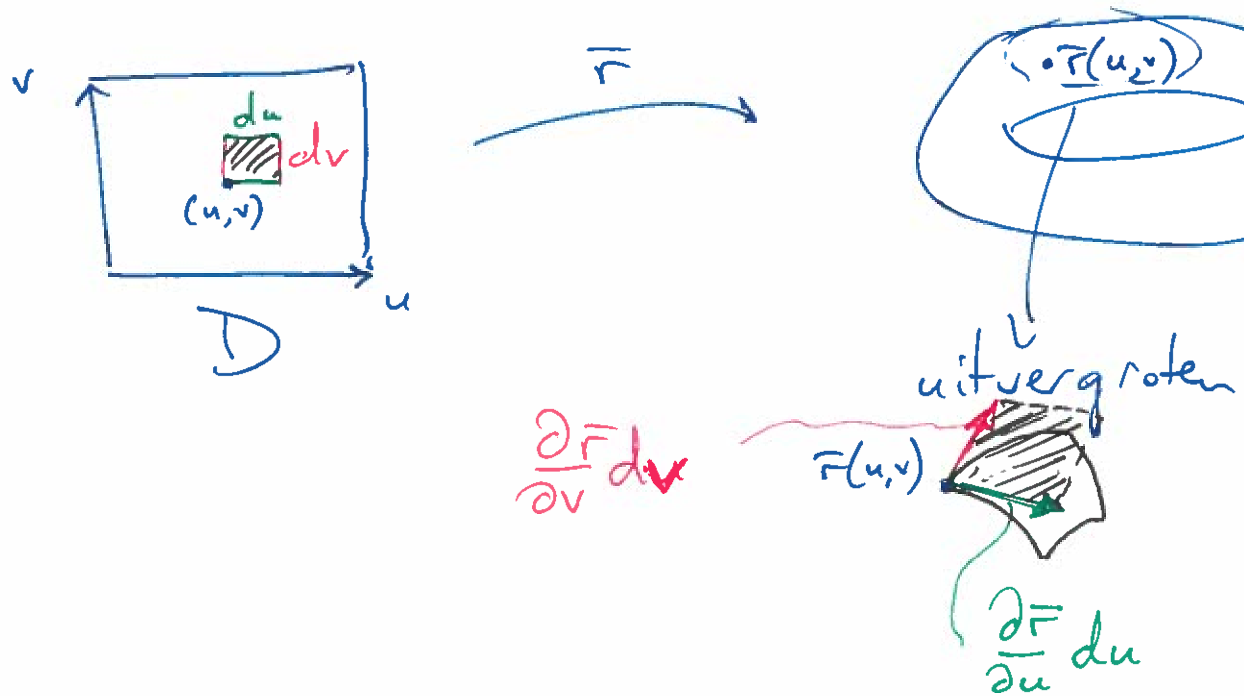
de flux van $\overline{F}$ door S is de integraal

$$\iint_S \overline{F} \bullet \underbrace{d\overline{S}}_{\text{vector variant van } dS} \stackrel{\text{per def.}}{=\!=} \iint_S \overline{F} \bullet \underbrace{\hat{N}}_{\text{unit normal vector}} dS$$

unit normal vector → staat loodrecht op het oppervlak S en heeft lengte 1

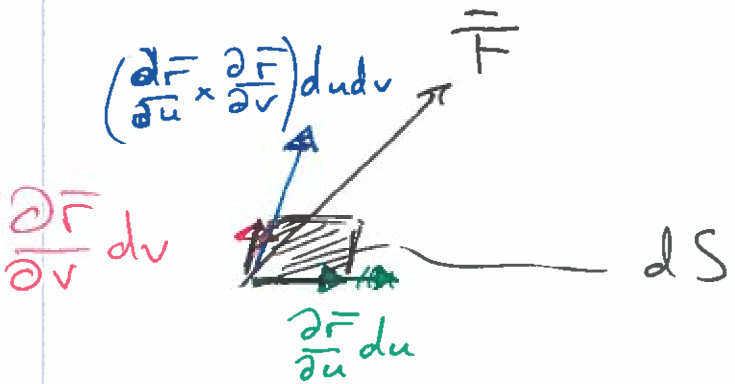
Het uitproduct $\frac{\partial \bar{r}}{\partial u} \times \frac{\partial \bar{r}}{\partial v}$

- lengte is gelijk aan het opp. met zijden $\frac{\partial \bar{r}}{\partial u}$ en $\frac{\partial \bar{r}}{\partial v}$ (van parallellogram
- ~~orthogonaal met S~~
- orthogonaal met $\frac{\partial \bar{r}}{\partial u}$ en $\frac{\partial \bar{r}}{\partial v}$
- drietal $\frac{\partial \bar{r}}{\partial u}, \frac{\partial \bar{r}}{\partial v}, \frac{\partial \bar{r}}{\partial u} \times \frac{\partial \bar{r}}{\partial v}$ is rechts georiënteerd.



$$\text{opp} = \left| \frac{\partial \bar{r}}{\partial u} du \times \frac{\partial \bar{r}}{\partial v} dv \right|$$

$$= \left| \frac{\partial \bar{r}}{\partial u} \times \frac{\partial \bar{r}}{\partial v} \right| du\, dv$$



Flux: $\bar{F} \cdot \hat{N}\, dS$ (defn).

unit normal vec

$$\frac{\frac{\partial \bar{r}}{\partial u} \times \frac{\partial \bar{r}}{\partial v}}{\left| \frac{\partial \bar{r}}{\partial u} \times \frac{\partial \bar{r}}{\partial v} \right|}$$

$$dS = \left| \frac{\partial \bar{r}}{\partial u} \times \frac{\partial \bar{r}}{\partial v} \right| du\, dv$$

vermenigvuldig

$$\frac{\partial \bar{r}}{\partial u} \times \frac{\partial \bar{r}}{\partial v}\, du\, dv.$$

Voorbeelde int. met/zonder scalarveld. $\iint_S f \, dS$

S = sfeer met straal R, $f = 1$ maw: wat is het opp van S

Neem bolcoord parametrisering:

$$\bar{r}(\vartheta, \varphi) = R(\cos\vartheta \sin\varphi, \sin\vartheta \sin\varphi, \cos\varphi)$$

D is: $0 \leq \vartheta \leq 2\pi$, $0 \leq \varphi \leq \pi$

$$\left|\frac{\partial \bar{r}}{\partial \vartheta} \times \frac{\partial \bar{r}}{\partial \varphi}\right| = R^2 \sin\varphi \geq 0$$

want $\frac{\partial \bar{r}}{\partial \vartheta} = R(-\sin\vartheta \sin\varphi, \cos\vartheta \sin\varphi, 0)$

$$\frac{\partial \bar{r}}{\partial \varphi} = R(\cos\vartheta \cos\varphi, \sin\vartheta \cos\varphi, -\sin\varphi)$$

$$\frac{\partial \bar{r}}{\partial \vartheta} \times \frac{\partial \bar{r}}{\partial \varphi} = R^2(-\cos\vartheta \sin^2\varphi, -\sin\vartheta \sin^2\varphi, -\sin^2\vartheta \sin\varphi\cos\varphi - \cos^2\vartheta \cos\varphi \sin\varphi)$$

$$= R^2 \sin\varphi \underbrace{(-\cos\vartheta \sin\varphi, -\sin\vartheta \sin\varphi, -\cos\varphi)}_{\text{punt op sfeer met straal } 1}$$

en $\sin\varphi \geq 0$ als $0 \leq \varphi \leq \pi$

$$\iint_S dS = \iint_D \left| \frac{\partial \bar{r}}{\partial \vartheta} \times \frac{\partial \bar{r}}{\partial \varphi} \right| d\vartheta\, d\varphi = \int_0^{\pi} \int_0^{2\pi} R^2 \sin\varphi \, d\vartheta\, d\varphi =$$

$$= 2\pi R^2 \int_0^{\pi} \sin\varphi \, d\varphi = 4\pi R^2$$