

Extra informatie ivm verslag 2

De wortel van een 2 bij 2 matrix

Laat

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \quad (1)$$

$$\text{en } B = A^{1/2} \quad (2)$$

Geval 1 : voor $a_{12} = a_{21} = 0$

Dan geldt:

$$B = \begin{pmatrix} \pm\sqrt{a_{11}} & 0 \\ 0 & \pm\sqrt{a_{22}} \end{pmatrix} \quad (3)$$

Geval 2: voor $a_{12}=0$ en $a_{21} \neq 0$ geldt:

$$B = \begin{pmatrix} \pm\sqrt{a_{11}} & 0 \\ \pm a_{21} / (\sqrt{a_{11}} + \sqrt{a_{22}}) & \pm\sqrt{a_{22}} \end{pmatrix} \quad (4)$$

Analoog krijgen we:

$$B = \begin{pmatrix} \pm\sqrt{a_{11}} & \pm a_{12}/(\sqrt{a_{11}} + \sqrt{a_{22}}) \\ 0 & \pm\sqrt{a_{22}} \end{pmatrix} \quad (5)$$

voor $a_{12} \neq 0$ en $a_{21} = 0$.

Geval 3: als $a_{12} \neq 0$ en $a_{21} \neq 0$

dan

$$B = \pm(1/R) \begin{pmatrix} (a_{11} + T) & a_{12} \\ a_{21} & (a_{22} + T) \end{pmatrix} \quad (6)$$

$$\text{waarbij } T = |A|^{1/2} = \sqrt{a_{11}a_{22} - a_{12}a_{21}} \quad (7)$$

$$\text{en } R^2 = a_{11} + a_{22} + 2T, R \neq 0 \quad (8)$$

Bewijs:

$$\begin{aligned}
B * B &= (1/R^2) \begin{bmatrix} (a_{11} + T) & a_{12} \\ a_{21} & (a_{22} + T) \end{bmatrix} \begin{bmatrix} (a_{11} + T) & a_{12} \\ a_{21} & (a_{22} + T) \end{bmatrix} \\
&= (1/R^2) \begin{bmatrix} (a_{11} + T)^2 + a_{12} a_{21} & a_{12}(a_{11}+T)+a_{12}(a_{22}+T) \\ a_{21}(a_{11}+T)+a_{21}(a_{22}+T) & (a_{22}+T)^2 + a_{12} a_{21} \end{bmatrix} \\
&= (1/R^2) \begin{bmatrix} (a_{11})^2 + 2 a_{11} T + (T^2+a_{12} a_{21}) & a_{12}(a_{11}+a_{22}+ 2T) \\ a_{21}(a_{11}+a_{22}+2T) & (a_{22})^2+2a_{22}T +(T^2+ a_{12} a_{21}) \end{bmatrix} \\
&= (1/R^2) \begin{bmatrix} (a_{11})^2 + 2 a_{11} T + a_{11} a_{22} & a_{12} R^2 \\ a_{21}R^2 & (a_{22})^2+2a_{22}T +a_{11} a_{22} \end{bmatrix} \\
&= (1/R^2) \begin{bmatrix} (a_{11}) R^2 & a_{12} R^2 \\ a_{21} R^2 & (a_{22}) R^2 \end{bmatrix} \\
&= A.
\end{aligned}$$

Voorbeelden:

(1) $A = \begin{bmatrix} 1 & 3 \\ 5 & 6 \end{bmatrix}$, $|A| = -9$, $T = 3i$ en $R^2 = a_{11}+a_{22}+2T = 1+6+6i = 7+6i$

$$B = [\pm 1/\sqrt{(7+6i)}] \begin{bmatrix} (1+3i) & 3 \\ 5 & (6+3i) \end{bmatrix}$$

(2) $A = \begin{bmatrix} 11 & -3 \\ 2 & 6 \end{bmatrix}$, $|A| = 72$ and $T = 6\sqrt{2}$, $R^2 = 17 + 12\sqrt{2}$

$$B = [\pm 1/\sqrt{(17+12\sqrt{2})}] \begin{bmatrix} 11+6\sqrt{2} & -3 \\ 2 & 6(1+\sqrt{2}) \end{bmatrix}$$