The validation of metaphors

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1 Introduction

In this paper we introduce models as metaphors for the description of reality. We consider a number of case studies of contemporary research - pollution of the North-Sea, the flow field of the Wadden Sea, drillstring dynamics, the use of metaphors in psychoanalysis. In all these cases validation of the results takes a very different form. Also this validation is certainly not in agreement with the picture of scientific research projected by textbook examples.

In the discussion we draw some conclusions on the use and appreciation of models and metaphors in the sciences.

2 Modeling reality

In this section we discuss the concept of a mathematical model and we point out the significance of metaphors and the part played by mathematical language. This leads to the question of validation of models and metaphors.

2.1 The mathematical model

In the natural sciences models of reality are usually mathematical objects which can be studied by mathematical methods. The results of such a study is then related again to reality. For instance a simple model like the quadratic difference equation (or logistic equation) models the restricted growth of a single population like an isolated bacteria culture in a laboratory or an isolated population of animals on an island. The predictions derived from analyzing this equation can be compared with observed time series of population growth. Another, rather classical example are Newton's equations of motion which can be used for instance to describe the dynamics of a system of particles. Analysis of these equations produces predictions of the motion of the particles, predictions which can be checked against observations.

The examples we mentioned are textbook examples of well-established models. They can be used to show that they mimick reality fairly accurately. This makes them also very deceptive. These examples are the outcome of a long process of thinking and testing, they are not typical for scientific modeling as it is carried out in daily practice. We return to this in a number of case studies in the next section.

Note also that, apart from models as mathematical objects, the term 'model' is being used in many other ways. A teacher may use the word model for constructions of polyhedra, a human skeleton or an organ. A biologist or a medical researcher may use a relatively simple, experimental laboratory set-up as a model for a complicated real-life phenomenon. The use of the word 'model' with so many different meanings can be confusing and it also may cause irritation.

The word 'model' as we shall use it in this paper, indicates a schematic representation of reality connecting the main quantities by laws which take the form of mathematical equations. In this formulation models are *mathematical* models; they pose qualitative questions, sometimes extremely difficult ones, and at the same time their nature is essentially quantitative. By producing figures and numbers we obtain a quantitative approximation of reality which is as accurate as possible.

Mathematical models are never to be confused with reality itself, they represent a very simplified view of an artificially isolated part of reality. A 'better model' means a better approximation of part of reality but reality itself is always something fundamentally different. Even in the natural sciences this causes major problems as concepts and ideas are changing in time. An old but interesting illustration is provided by the phenomenon 'electron'. Most people think of an electron as a very small particle with an electric charge. Such a particle may orbit atomic nuclei or many electrons may flow collectively as a current. In this way however, the indication 'electron' is more like a metaphor than a model of reality. James Gleick (1992) gives a transparent description of this struggle with language in physics:

> "The atom of Niels Bohr, a miniature solar system, had become an embarrassingly false image. In 1923, on the tenth anniversary of Bohr's conception, the German physicist Max Born hailed it: "the thought that the laws of the macrocosmos in the small reflect the terrestrial world obviously exercises a great magic on mankind's mind" - but already he and his colleagues could see the picture fading into anachronism. It survived in the language of angular momentum and spin - as well as in the standard high-school physics and chemistry curriculums - but there was no longer anything plausible in the picture of electrons orbiting a nucleus. Instead there were waves with modes of resonance, particles that smeared out propabilistically, operators and matrices, malleable spaces with extra dimensions, and physicists who forswore the idea of visualisation altogether."

Still, until today the metaphor of an electron as a small particle with a charge has its restricted use in handling the abstract concepts.

2.2 Metaphors

What is a metaphor and why do we find them useful? Using a metaphor, as Aristotle (ed. 1985) said, "consists in giving the thing a name that belongs to something else". It is impossible to describe and to theorize about new things without referring to well-known things. We can only understand or place something if it is or seems to be like something we know already. Even physicists who are using all the abstract characteristics of an electron, still find it helpful to think of it as a small particle with a charge. It turns out that in scientific concept formation and discussion metaphors are unavoidable.
This persistent use of metaphors even holds for many concrete things as we may observe in the language of computer use. The computer has a 'memory', it 'reads', 'stores', 'uses a language' etc. Actually it does none of these things literally but it helps to use the terms.

As we have seen, a scientific model is not reality itself, it is a description of reality and in fact again a metaphor. A model however is a metaphor with something added, it has not only qualitative but also quantitative aspects which adds to the precision of description.

We used the expression 'approximation of reality which is as accurate as possible'. This implies the validation of the model. In the next section we shall see that modeling and validation can take many different forms.

2.3 The language called mathematics

In the sciences mathematics plays a special part as it is not a natural science but one of the humanities handling pure imaginary objects. Although famous for its way with numbers - the quantitative side - mathematics is really about structures and the deeper relations between mathematical objects, the qualitative side. The solvability of an equation and its relation with other equations is often more interesting to the mathematician than the solution itself.

In mathematics moreover, images are created which are really new and which are an inspiration for metaphoric thinking, in particular modeling. A classical example are the regular polyhedra which inspired Kepler to develop his view of the cosmos. Recent examples are the images of fractals and strange attractors which inspire modeling in many fields of application.

Chaotic Dynamics

In the cases which we shall discuss below, we need a number of ideas and concepts of chaotic dynamics. Although this is a rather technically complicated branch of mathematical dynamical systems theory it is possible to give an idea of the key concepts and consequences. To simplify the discussion in the next sections, we summarize a number of these ideas here. For nice introductions see Stewart (1990), Peitgen et al. (1992) and (in Dutch) Broer et al. (1995), Broer and Verhulst (1992).

In chaotic dynamics we have evolution in time of a number of state variables, an evolution which is given by deterministic laws which take the form of mathematical equations. The governing laws are deterministic, there are no stochastic elements in the equations and there is no stochastic input in the system. In a large number of such systems we can detect chaos which is characterized as follows:

- For most of the initial states the evolution in the system is bounded but does not converge towards a regular type of state like equilibrium or a periodic solution.
- There is extremely sensitive dependence on the initial state in the sense that when we start two evolutions which are initially very close, after some time they will always separate widely.
- In a large number of examples the evolution in a deterministic chaotic system seems to take place near a 'strange attractor', a geometrical structure which is of fractal nature.
- Near such a strange attractor the evolutions are both attracted and repelled. At the same time the evolutions cannot leave a neighbourhood of the attractor which results in chaotic behaviour.

One should consult introductory books to obtain a visualization of this exciting kind of dynamics. Famous examples are the equations of population dynamics - the logistic equation is one - and the equations studied in meteorology, for instance the Lorenz equations with its butterfly-shaped strange attractor. In the natural sciences chaotic dynamics puts a severe restriction on predictability in a large number of problems. In meteorology one talks for instance about the 'predictability horizon' which puts an absolute limit to valid weather predictions, a limit which is measured in days or weeks and not longer.

There are applications of chaotic dynamics to many other fields like fluid mechanics, astrophysics, mechanical engineering, economics, fysiology, solar system dynamics etc. It should be noted that chaos is not without an underlying structure which may vary from case to case. In research these signatures of chaos (bifurcation sequences, fractal dimensions etc.) are established. At the same time interpretation of data and validity of models is very different and rather difficult because of the lack of predictability of the phenomena.

Mathematical language is the tool of modern science; its consistent use, together with the unsparing rule of experimental verification, started off modern science. However, it is an open question why mathematics, studying the laws of imaginary objects in an imaginary world, is so unusually effective in modeling reality. This must have something to do with the way we perceive the world and the corresponding pattern formation which plays a part in wiring the human brain. How it actually works is not clear.

2.4 Validation

A scientific model is a metaphor with quantitative aspects added. Ideally the validation of such models requires setting up carefully controlled experiments which allow for measurements to be matched with the quantitative predictions of the model, the outcome of mathematical calculations. Again, a famous example is formed by Newton's equations of motion describing the statics and dynamics of particles and bodies.

In practice 'carefully controlled experiments' are often difficult or virtually impossible to achieve. In biology, in astrophysics, even in classical mechanics natural phenomena arise which cannot be controlled. The set of data which is necessary for validation is incomplete, partly because of deficient measuring techniques but also because of some form of inaccessibility of the phenomena. In the sequel we shall discuss some examples.

It is clear that in these problems the validation of the model takes place in a different way. The measurements are matched with the quantitative predictions as far as possible but there is strong emphasis on qualitative aspects: do patterns arise in the model which have been actually observed, are the time-scales of the phenomena more or less correct, are global quantities like energy output or mass transport correctly predicted etc. This type of validation is different from what we are used to in classical physics.

The use of metaphors with qualitative aspects only is essential to concept formation and the development of new ideas in some fields. In this respect we shall discuss an example of an application of nonlinear dynamics in psychoanalysis. Other interesting new applications are derived from mathematical 'game theory' - with the wellknown example of the 'Game of Life' - and the theory of 'cellular automata' used in artificial cognitive systems.

Making sense of such metaphors will take a different shape in each field of application. A metaphor is validated if people agree that it introduces a pattern or dynamics which adds a new element towards understanding and explaining processes and phenomena in the field of application. So validation of a metaphor contains at least two elements, the possibility of peer review by researchers in the field and a consistent view on the concept of 'explanation' in this field of application.

Both elements are nontrivial. The possibility of peer review requires open minds, a nonsectarian attitude and careful use of authority by leading experts; for a discussion on psychoanalysis in this respect see for instance Mitchell (1998).

The concept of explanation will take very different forms in different disciplines. Mechanical, historical, philosophical or psychoanalytical explanations for instance are all very different concepts. However in all cases they have to do with giving meaning to the phenomena studied in a discipline. For a well-balanced view by an outsider of the development of ideas and explanation in psychoanalysis see Nagel (1995). Discussions in the framework of the social sciences were given by Hayek (1955) and de Groot (1961).

We shall illustrate validation problems in our case studies.

3 Case studies

In this section we shall review a number of concrete modeling problems in science and their validation. It turns out that the practice of scientific work is far removed from the textbook examples mentioned earlier.

Now and then we shall use some barbaric technical terms. We are hoping that the reader who does not know these terms will read on and will still obtain the general idea.

3.1 Pollution of the North-Sea

The North-Sea is a tidal basin in which continuous transport of polluting chemicals takes place. In such a bounded sea basin we have a time-periodic flow field which is caused by the tides. Also there is a continuous input of riverwater from all the surrounding countries, water which is polluted by the chemical industry, by farmers and by waste-deposits from the cities along the rivers.

The mathematical model of this transport of chemicals in the sea takes the form of a nonlinear parabolic differential equation with initial and boundary conditions. What we want to know is the concentration of chemicals as a function of time and place; this is the solution of the differential equation. In the model we have, apart from the tidal force field and the river-input, advective transport of chemicals, small diffusion of chemicals and a reaction term which involves settlement of chemicals on the sea floor and reactions in the water itself. For details see Krol (1990, 1991).

The time-scales which play a part in this model are determined by the tides - about twice a day - and the slow diffusion of chemicals in the water, which from its disposal into the sea needs roughly nine months to reach near-equilibrium.

The model requires advanced mathematical techniques and produces quantitative predictions about the spreading and concentration of chemicals in the North-Sea. It turns out that all the river-inputs are mixing and are initially spreading out over the North-Sea, also there is a reststream along the Dutch coast which pollutes the Dutch and German Wadden Sea. Most of the chemicals end up in the north-east part of the North-Sea.

There are a number of uncertainties in the model, for instance the form of the reaction term and the importance of assumptions about the geometry of the basin.

Validation

What about validation of the model? Theoretically the mathematical equations are well founded except for the term which describes the reaction of chemicals with sea water and bottom. On the other hand changing this term somewhat does not change the predictions dramatically which gives credibility to the model.

Empirically: we cannot follow individual particles coming out off the rivers, also we cannot measure the influence of an isolated river. It is hardly possible to do experiments but we can measure concentrations at the places where the rivers enter the North-Sea and at places in which we take a special interest like the Wadden Sea and the north-east part.

Validation of the model is very restricted and takes place in a global way: the measurements confirm that the chemicals are spreading out over the North-Sea, there is an increased concentration in the north-east and the relevant time-scale of this spreading is roughly nine months. This confirmation is global because we cannot follow individual particles and the detailed process of mixing in place and time; so this confirmation cannot be considered as a detailed quantitative validation of the model. However the model is used to understand and to make predictions about what is going on regarding pollution in the North-Sea.

3.2 The chaotic flow-field of the Wadden Sea

The Wadden Sea is a tidal basin behind a chain of islands in front of the Dutch, German and Danish coast. The dynamics of the flow-field in this shallow basin is mainly determined by the tides which enter from the North-Sea twice a day and by the sandplates and channels which show strong geomorphological complexity.

It is important to have theoretical insight in the nature of the flow-field as this is tied in with sediment transport which in its turn determines the existence and safety of the islands and shipping lanes. Computer simulations produce results which are very sensitive to initial conditions and the changing geomorphology. However, it has been recognized (see Ridderinkhof and Zimmerman, 1992) that the flow patterns are all of them examples of two-dimensional conservative dynamics which is characterized by flow in regular bands and between the bands chaotic displacement of the water. In the calculations these places of chaotic stirring can be identified and compared with measurements of local stream velocities in the Wadden Sea.

Validation

The first step in this important piece of research is the identification of the flow-field of the Wadden Sea as an example of conservative chaotic dynamics. This theoretical identification for flow of water is widely accepted and conservative dynamics has become a well-researched and established part of dynamical systems theory; its results can be trusted.

Qualitatively the actual application of the theory to a basin with such a complicated geometry is possible as we know that the equations of conservative chaotic dynamics apply. The implication is that the flow-field of the Wadden Sea should have the typical patterns of this dynamics which are centre circulation points and bands of chaotic motion.

A quantitative assessment is very difficult. The outcome and the predictions for the flow-field are extremely sensitive to initial conditions, especially where chaotic stirring - the most interesting phenomenon - takes place.

So qualitatively this approach is in a strong position as the predicted typical patterns do arise but to make predictions which can be checked against measurements is hardly possible. What one can do quantitatively is to indicate the locations of chaotic stirring and to check whether the measured flow-field fits with the signature of chaos typical for conservative dynamics. This can be seen as a global test and it turns out to yield such positive results that the theory of a chaotic flow-field in the Wadden Sea is considered to be correct.

3.3 Drillstring dynamics

Geophysical drilling or deep well drilling for oil or gas takes place by using a rotating bit to generate a borehole and a drillstring to rotate the bit. Mud is continuously pumped up and down to cool and lubricate the bit and to transport the cuttings to the surface. Drilling depths of 5 km are not unusual, for geophysical drillings one even reaches a depth of 10 km.

One of the main problems is that there are repeatedly breakdowns which make it necessary to extract the drillstring and bit to replace some parts. This extraction is laborious and it is effectively one of the main causes of the high cost of drilling.

To understand what is going on deep down one cannot have recourse to observations. Especially at a breakdown all devices attached to the drillstring for measurements are destroyed. The only way to obtain insight is simulation by constructing a mathematical model and analyzing this. A model for a rotating bit has been constructed by Jansen (1991, 1993), a mathematical analysis has been given by van der Heijden (1993, 1994). The results can be summarized as follows.

Increasing the speed of the drillstring the bit displays certain periodic movements which at a special speed become chaotic. Lower speeds are not efficient to carry out the drilling and to escape from chaos one has to go on to unrealistic high values of the rotation speed. The analysis by van der Heijden (1994) of a relatively simple model shows a very complicated chaotic behaviour with many interesting theoretical aspects.

Validation

The mathematical model built for the rotating bit is based on well-known laws of classical mechanics. The choice of the number of dimensions of the system (degrees of freedom) is as low as possible while still capturing an essential part of the dynamics. Apart from the breakdown of the machinery at a certain rotation speed we have no quantitative information or measurements.

The mathematical model in itself is highly sophisticated but as a model for drillstring rotation it is a metaphor only as the information we can relate to reality is almost purely qualitative. We conclude that the rotating bit will behave chaotically at certain rotation speeds and that the device will probably break down at some point. Although the mathematical analysis is hard, the results from an engineering point of view are fairly poor.

Still we have to be satisfied with such results as there seems to be no other way out to understand what is going on.

3.4 Metaphors for psychoanalysis

Psychoanalysis is a field where qualitative considerations are far more important than quantitative analysis. In discussing the world of the mind, of feelings, the use of metaphors is a powerful way to obtain more insight as it gives a name to the unknown by comparing it with wellknown things.

Many metaphors in psychoanalysis - and in psychology - have been derived from basic mechanics. For instance 'the displacement of a problem', 'attachment to someone', 'a stable person', 'a split personality', 'confused state'. Some of these expressions have become so colloquial that people have forgotten that they are metaphors.

Recently one has realized that psychoanalysis may gain from the use of metaphors derived from new developments in science, in particular from chaotic dynamics; see for instance Moran (1991).In this view the human mind is considered as a dynamical system. A patient who is being treated has a state of mind experiencing many interactions: between the patient's emotions and feelings, verbal associations, the presence and the interventions of the psychoanalyst, the physical experience of the surroundings etc. These interactions, together with the influence of conscious and unconscious memory, cannot simply be added linearly to describe the mental state of the patient. They influence each other and together they determine the state of a dynamical system. There is also a strong and sometimes sensitive dependence on initial values: these are the childhood experiences which determine to a great extent the present in a deterministic, but, because of the complexity, not in a predictable way.

The similarities between mental processes and nonlinear dynamical systems go a long way. We discuss an example from Verhulst (1994).

An adult human being has a fixed collection of unconscious fantasies about himself and his environment. If the human mind is described (metaphorically) as a dynamical system, this collection of fantasies imbedded in the unconscious plays the part of a strange attractor. It is possible that only a small part of this strange attractor is active, i.e. one small part of the attractor is frequented often by the solutions of the dynamical system and this may cause disorders like compulsive repetition of actions.

For instance a patient can have the urge to actions like frequent hand washing. This may be a reaction to strong internal conflicts, for instance arising from a strong aggressive impulse, the anxiety about this and the ban on this agressive impulse. The anxiety takes here the form of hosophobia and frequently washing hands is "the solution". The unconscious fantasy is both attractive and frightening and so repelling. The symptom, frequent hand washing, may in this way be the outcome of contradictory and conflicting tendencies. The force of this process causes lack of freedom and lack of space in personal life.

In the terminology of chaotic dynamics we have that this strange attractor contains a small subset which is visited relatively often by the solutions (evolutions). Without intervention the process will continue 'forever'. What we could try for the patient in terms of therapeutic interventions is to change by well-timed perturbations the coupling between this active part and his mind as a whole. In this way the behavioural pattern could change, psychoanalytic therapy may result in a higher level of complexity and less rigidity.

Validation

This use of a metaphor from nonlinear dynamics and the possibility of corresponding therapies is too recent to be fully explored. The use of new language, terminology, is in itself a creative process which may trigger new ideas and concepts; this is stressed in Verhulst (1994).

As stated in section 2 the metaphor is validated if psychoanalysts agree among themselves (peer review) that this dynamics adds an element towards understanding the mental process and that it gives meaning to the phenomena and the corresponding therapies. But it should be noted that this is far removed from the systematic testing of hypotheses in consulting rooms. Such type of validation makes no sense in psychoanalysis.

4 Discussion

The purpose of this note on the validation of metaphors has been to argue that all scientific models are metaphors and that the ratio of qualitative to quantitative elements in scientific modeling may vary strongly.

Strangely enough, in discussions on validation in the social sciences philosophers of science play an important part; a well-known example is Popper (1963, 1968). An explanation for this is that apart from the emphasis on methodology in the philosophy of science the conclusions there are rather apologetic. For instance Popper: "A theory is scientific if it rules out, forbids the occurrence of some events" and "Don't look for verification, look for crucial tests which may refute the theory". In his view bodies of knowledge and doctrine like marxism and psychoanalysis are unscientific as they do not pass the test of falsifiability which is implicit in the above statements.

In fact Popper's requirements are correct, they have been in use from the dawn of modern science. Scientific theory is complete (for the time being) when tested thoroughly against reality by predictions and experiments. However, as has also been noted by Hayek (1955), in this strict sense it applies to textbook examples only, to classical examples of the history of science. It is typical that Popper's examples have been derived from mechanics, a field with centuries of development. As we have seen in our case studies, in contemporary science this ideal is never reached. Even stronger: problems where this ideal is reached are not fundamental and not of interest to the scientist.

The four case studies which we have presented are examples on a gliding scale. In the first case, the pollution in the sea problem, quantitative elements in the model and its validation are still rather dominant although not exclusively so. Going on to the other cases the quantitative element becomes weaker, the qualitative element becomes more prominent.

It would be easy to supplement our case studies by many other examples from topics like epidemiology, astrophysics, theoretical biology, climate research. If we would do that we would find that in all these subjects the criteria for the credibility of new scientic theories are different and changing in time, as befits organically growing systems. This would be an interesting subject of study in itself.

Fast growing disciplines like economics, sociology, psychology are, with regards to their modeling performance, similar to contemporary research in the natural sciences.

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